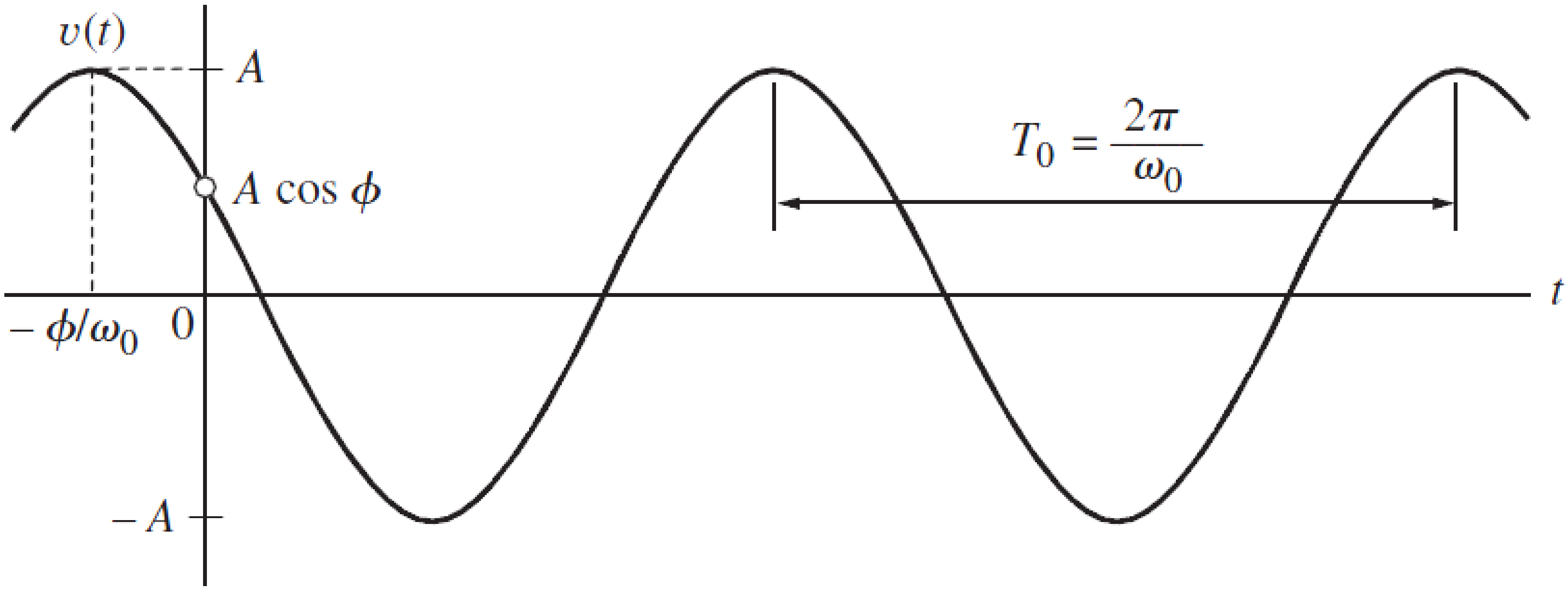
Unidad 2: SEÑALES ELÉCTRICAS EN EL DOMINIO DE LA FRECUENCIA:

Series de Fourier. Trigonométrica y exponencial

Transformada de Fourier. Teorema de Parseval.

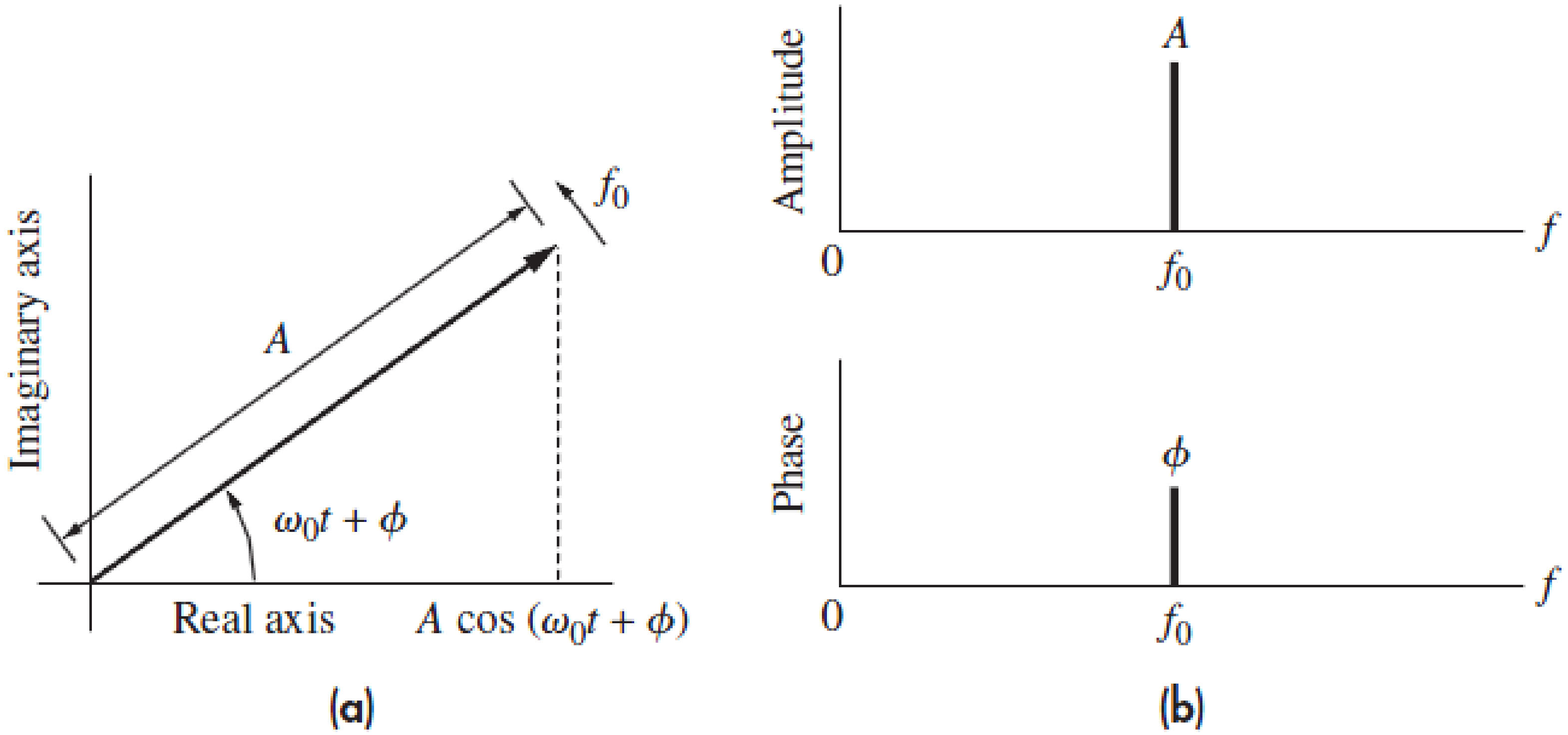
Espectros de densidad de potencia/energía. Teoremas relacionados con la Transformada de Fourier. Delta de Dirac, propiedades, aplicaciones. Espectro de señales periódicas. La transformada discreta de Fourier. Señales aleatorias en dominio de frecuencia. Espectro de densidad de potencia. Función de autocorrelación. Señales de banda angosta, características y modelado.

* Cap. 2 Carlson-Crilly



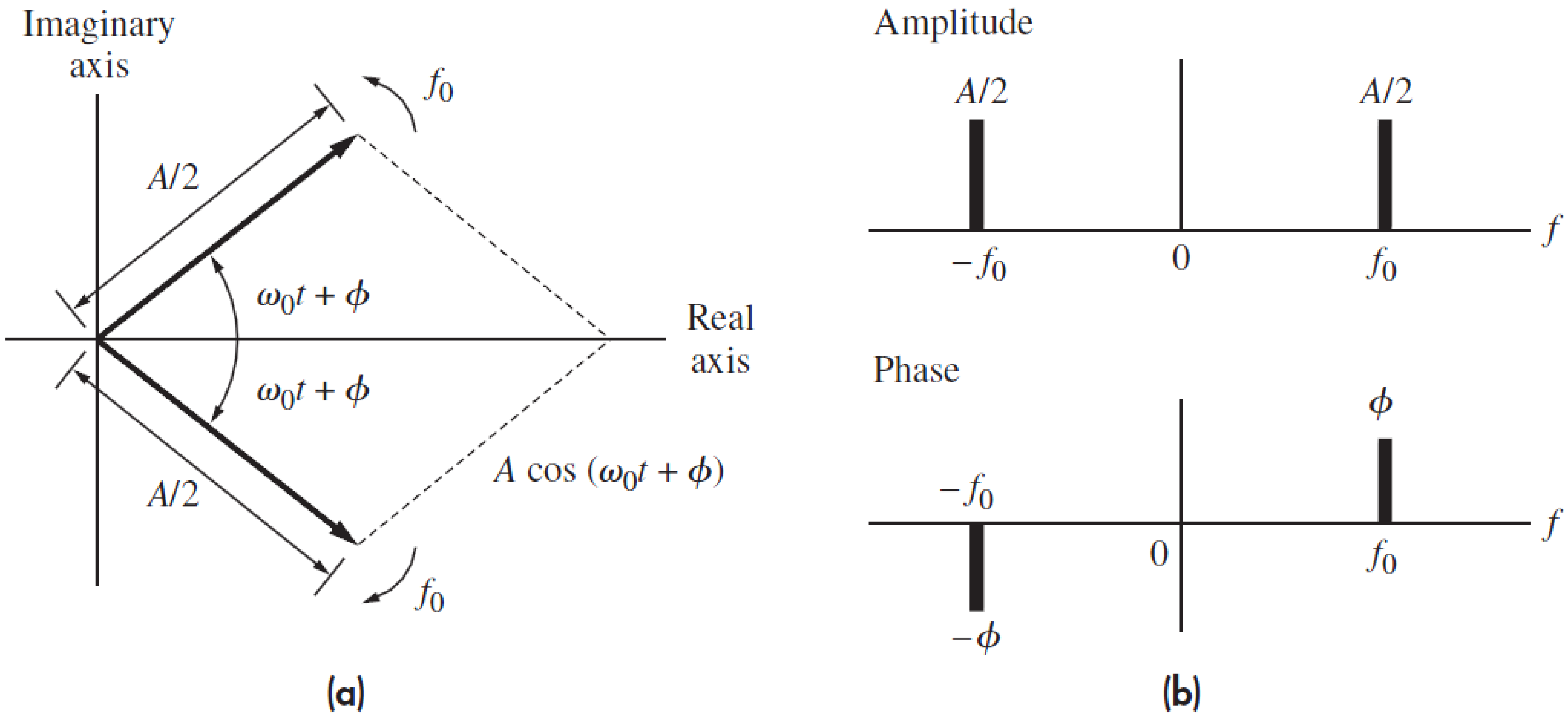
A sinusoidal waveform $v(t) = A \cos(\omega_0 t + \phi)$.

$$A \cos(\omega_0 t + \phi) = A \operatorname{Re}\left[e^{j(\omega_0 t+\phi)}\right]$$
$$= \operatorname{Re}\left[A e^{j\phi} e^{j\omega_0 t}\right]$$



Representations of $A \cos(\omega_0 t + \phi)$: (*a*) phasor diagram; (*b*) line spectrum.

$$A\cos(\omega_0 t + \phi) = \frac{A}{2}e^{j\phi}e^{j\omega_0 t} + \frac{A}{2}e^{-j\phi}e^{-j\omega_0 t}$$



(*a*) Conjugate phasors; (*b*) two-sided spectrum.

Series de Fourier

$$v(t) = \sum_{n=-\infty}^{\infty} c_n e^{\,j2\pi n f_0 t} \qquad n = 0, 1, 2, \ldots$$

$$c_n = \frac{1}{T_0}\int_{T_0} v(t) e^{-j2\pi n f_0 t}\, dt$$

$$c_n = \left|c_n\right| e^{j \arg c_n}$$

$$c_n\, e^{j2\pi n f_0 t} = \left|c_n\right| e^{j \arg c_n} e^{j2\pi n f_0 t}$$

Series de Fourier

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$$c_n = |c_n|\, e^{j \arg c_n}$$

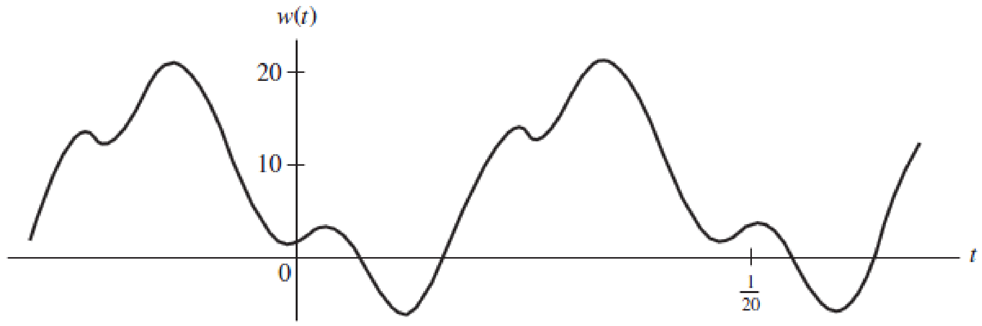
$$c_n\, e^{j2\pi n f_0 t} = |c_n|\, e^{j \arg c_n} e^{j2\pi n f_0 t}$$

V(t)?

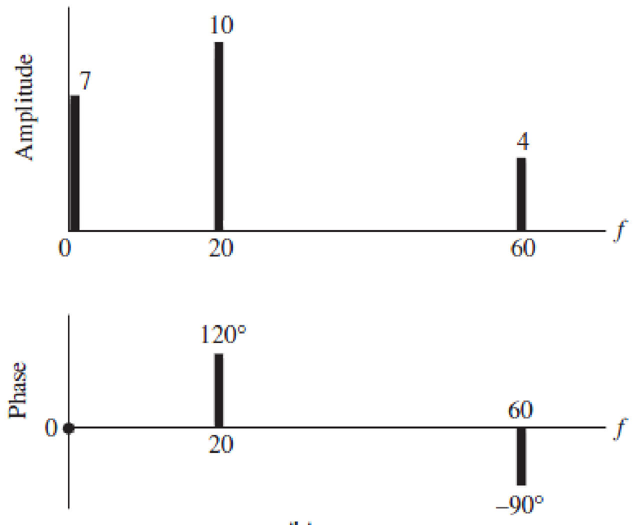
Series de Fourier

$$w(t) = 7 - 10\cos(40\pi t - 60°) + 4\sin 120\pi t$$

$$c(0) = \frac{1}{T_0}$$

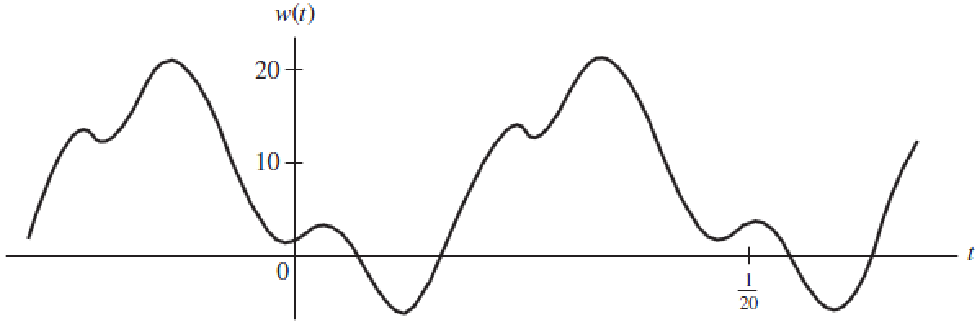


(a)

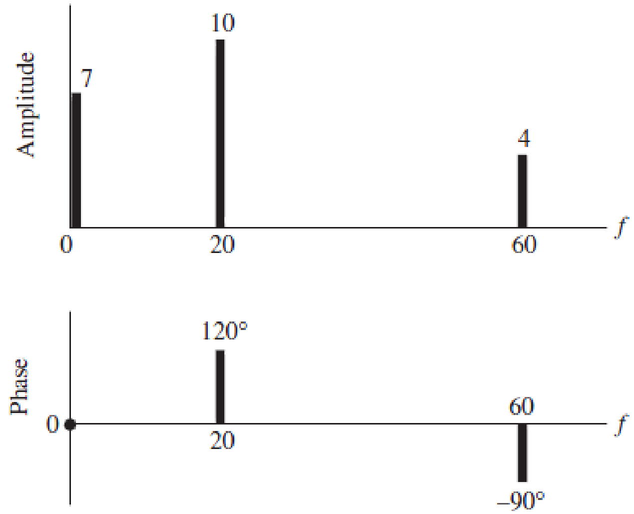


(b)

Series de Fourier



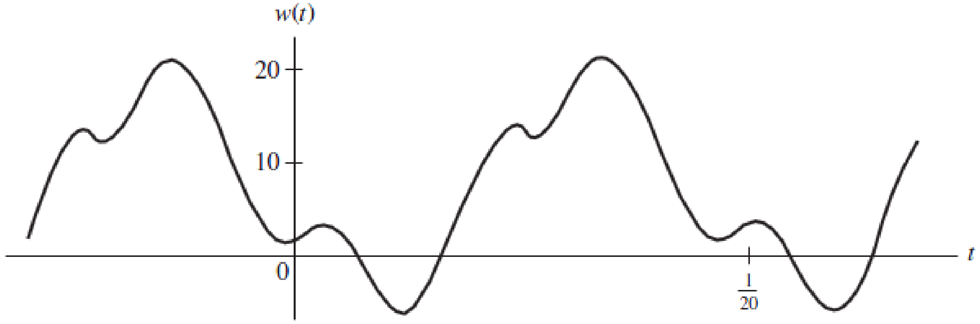
(a)



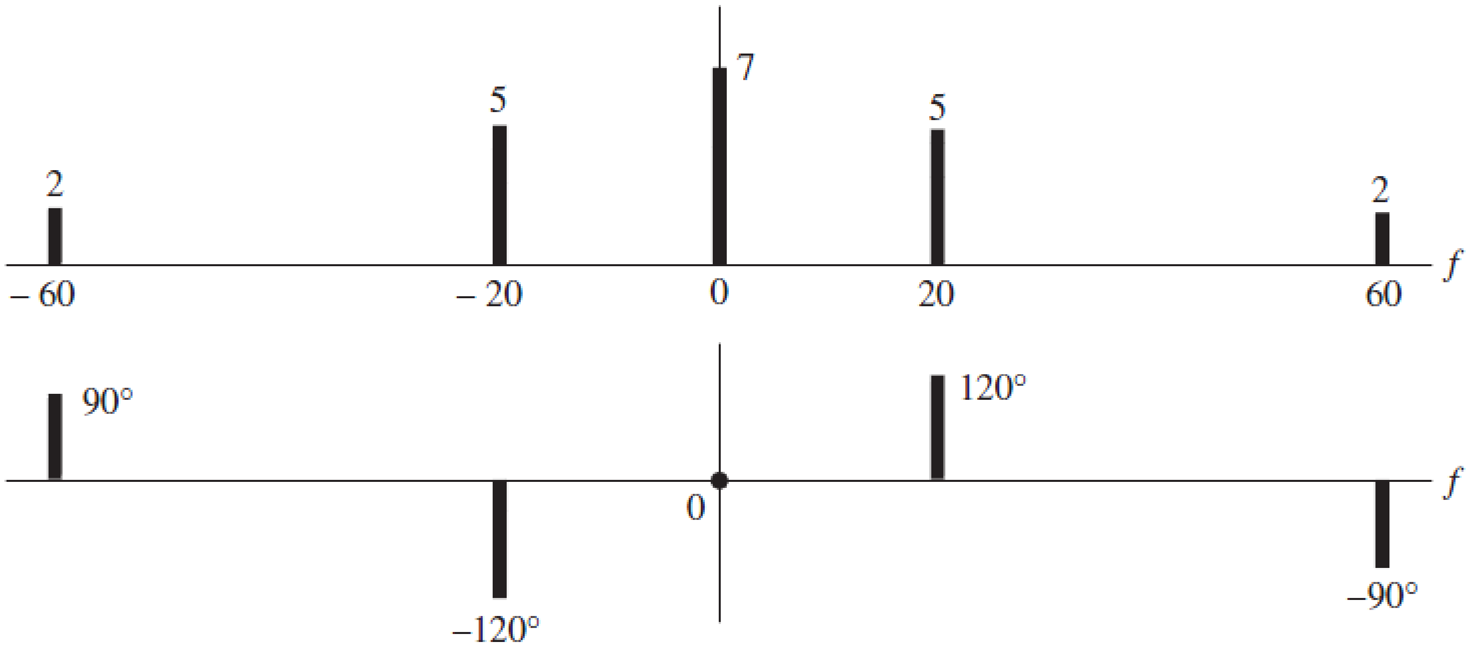
(b)

$$c(0) = \frac{1}{T_0}\int_{T_0} v(t)\, dt = \langle v(t)\rangle$$

Series de Fourier



(a)



Series de Fourier

Exponencial

$$v(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t} \qquad n = 0, 1, 2, \ldots$$

$$c_n = \frac{1}{T_0} \int_{T_0} v(t) e^{-j2\pi n f_0 t} \, dt$$

Trigonométrica

$$v(t) = c_0 + \sum_{n=1}^{\infty} |2c_n| \cos(2\pi n f_0 t + \arg c_n)$$

$$v(t) = c_0 + \sum_{n=1}^{\infty} [a_n \cos 2\pi n f_0 t + b_n \sin 2\pi f_0 t]$$

Otras formas?

Series de Fourier

Exponencial

$$v(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t} \qquad n = 0, 1, 2, \ldots$$

$$c_n = \frac{1}{T_0} \int_{T_0} v(t) e^{-j2\pi n f_0 t}\, dt$$

Trigonométrica

$$v(t) = c_0 + \sum_{n=1}^{\infty} |2c_n| \cos(2\pi n f_0 t + \arg c_n)$$

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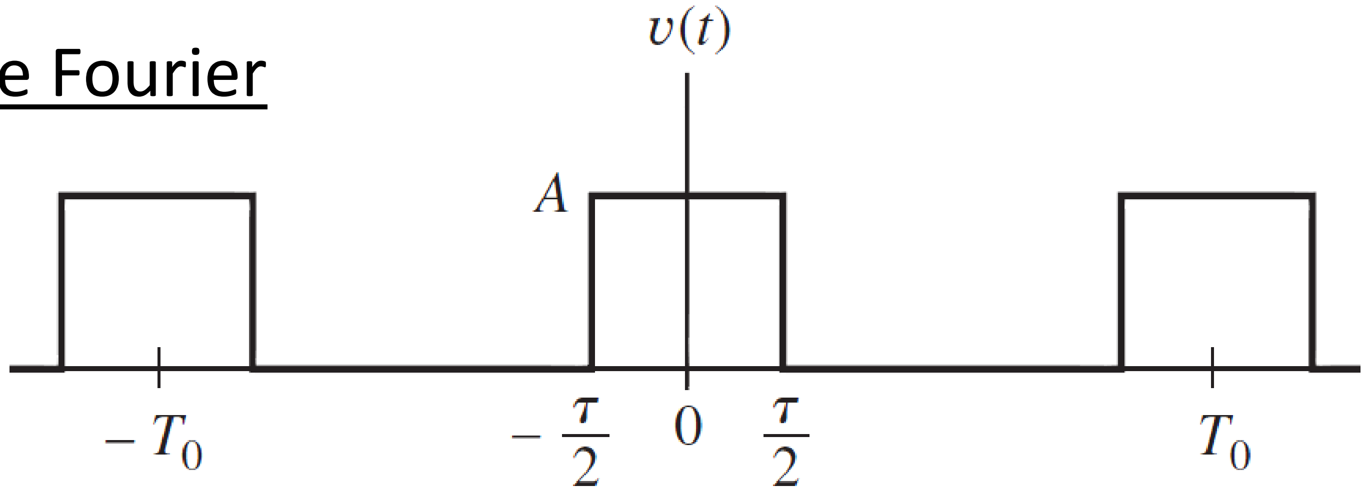
Series de Fourier

Ejemplo



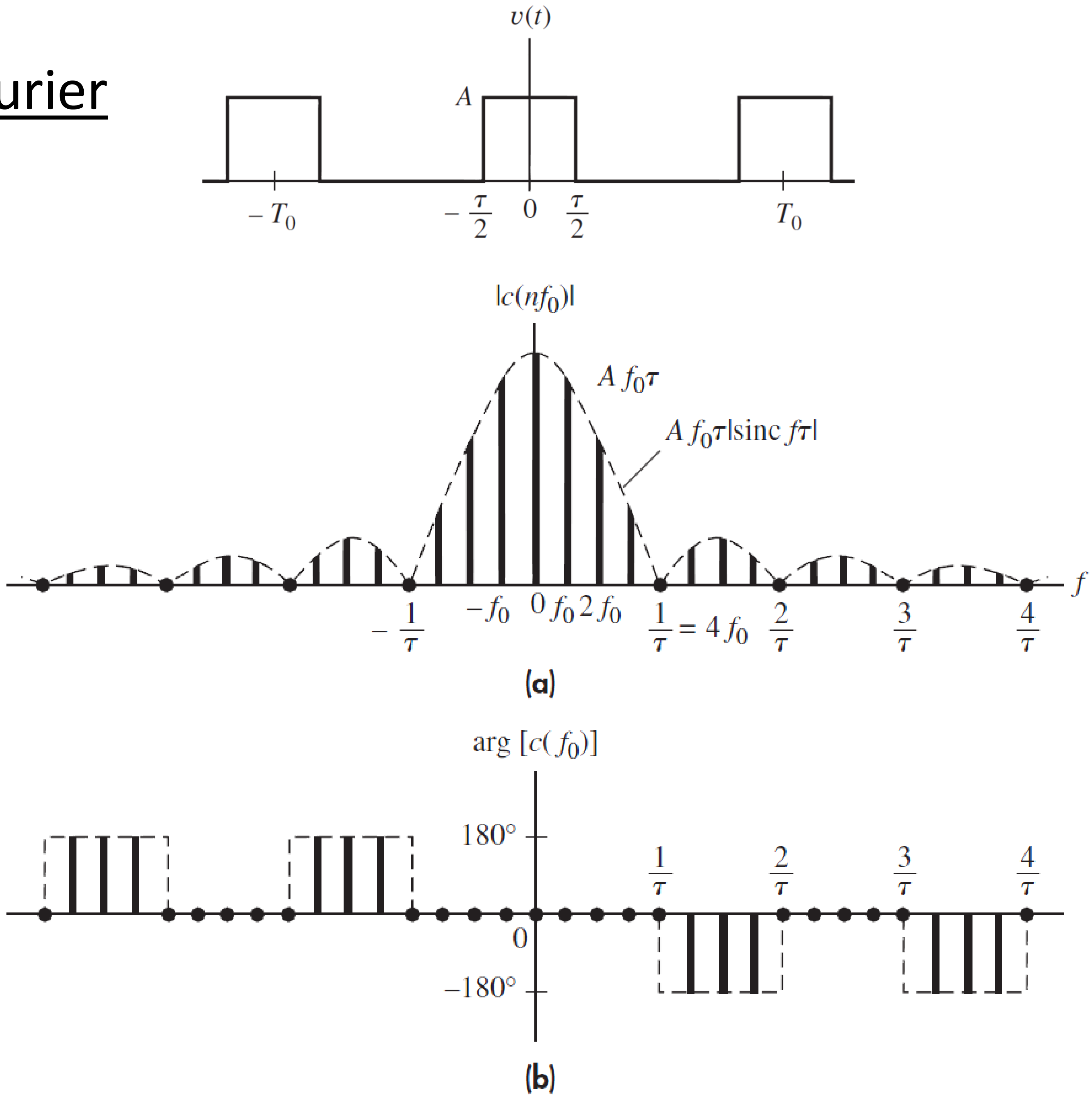
$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} v(t)e^{-j2\pi n f_0 t}\,dt = \frac{1}{T_0}\int_{-\tau/2}^{\tau/2} Ae^{-j2\pi n f_0 t}\,dt$$

$$= \frac{A}{-j2\pi n f_0 T_0}\left(e^{-j\pi n f_0 \tau} - e^{+j\pi n f_0 \tau}\right)$$

$$= \frac{A}{T_0}\frac{\sin \pi n f_0 \tau}{\pi n f_0}$$

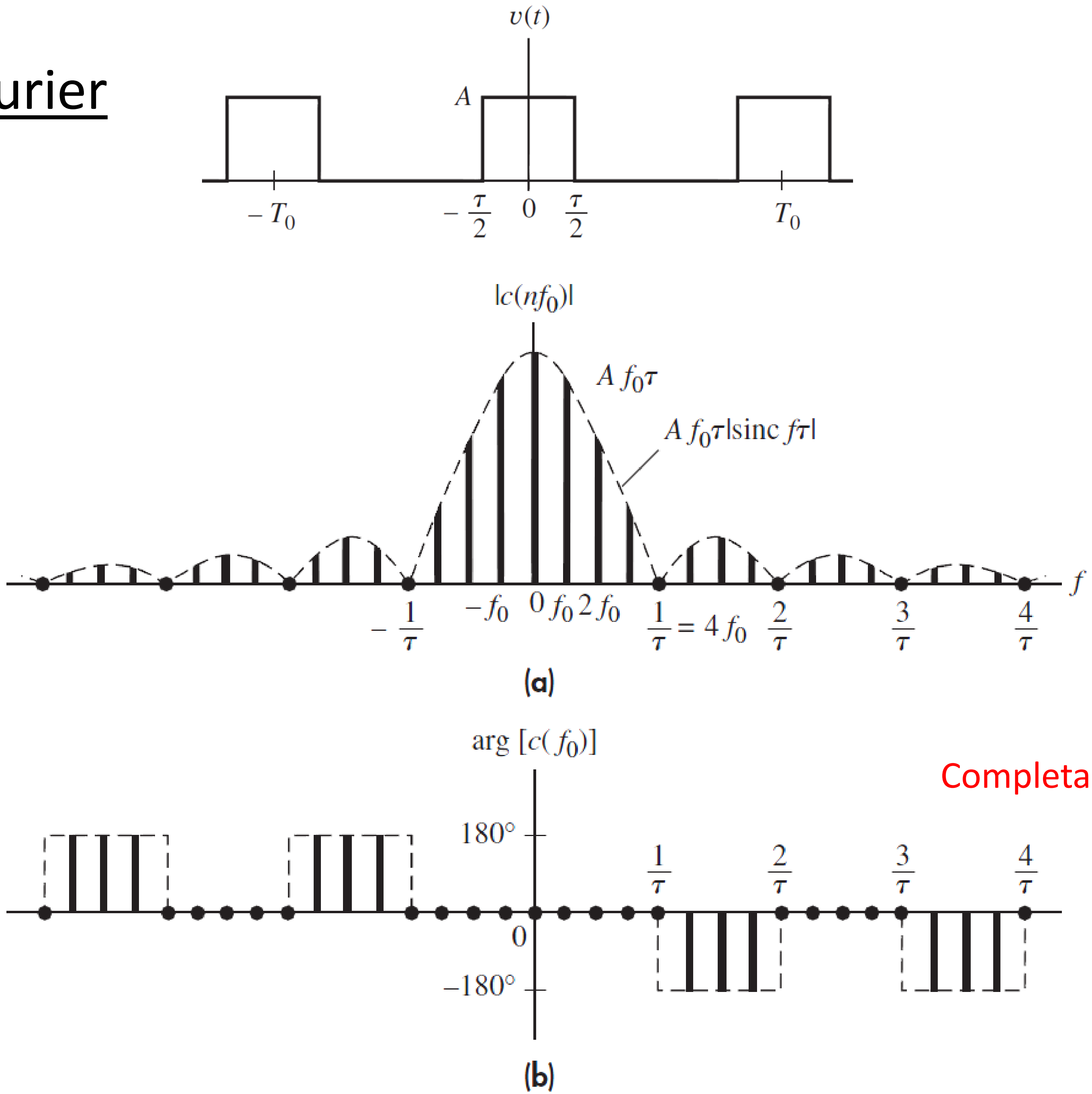
$$c_n = \frac{A\tau}{T_0}\,\mathrm{sinc}\, n f_0 \tau$$

Series de Fourier



Spectrum of rectangular pulse train with $f_c\tau = 1/4$. (*a*) Amplitude; (*b*) phase.

Series de Fourier



Spectrum of rectangular pulse train with $f_c \tau = 1/4$. (*a*) Amplitude; (*b*) phase.

Completar…

Teorema de Parseval

$$P = \frac{1}{T_0}\int_{T_0} |v(t)|^2\, dt = \frac{1}{T_0}\int_{T_0} v(t)v^*(t)\, dt$$

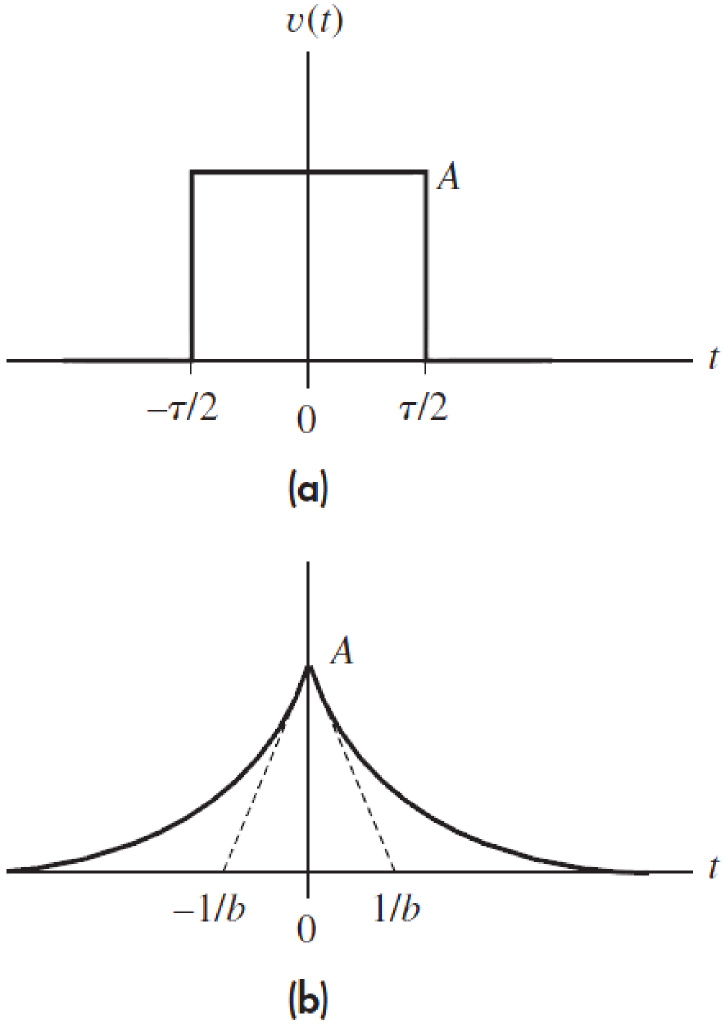
$$v^*(t) = \left[\sum_{n=-\infty}^{\infty} c_n\, e^{\,j2\pi n f_0 t}\right]^* = \sum_{n=-\infty}^{\infty} c_n^*\, e^{-j2\pi n f_0 t}$$

$$P = \frac{1}{T_0}\int_{T_0} v(t)\left[\sum_{n=-\infty}^{\infty} c_n^*\, e^{-j2\pi n f_0 t}\right] dt$$

$$= \sum_{n=-\infty}^{\infty}\left[\frac{1}{T_0}\int_{T_0} v(t)e^{-j2\pi n f_0 t}\, dt\right] c_n^*$$

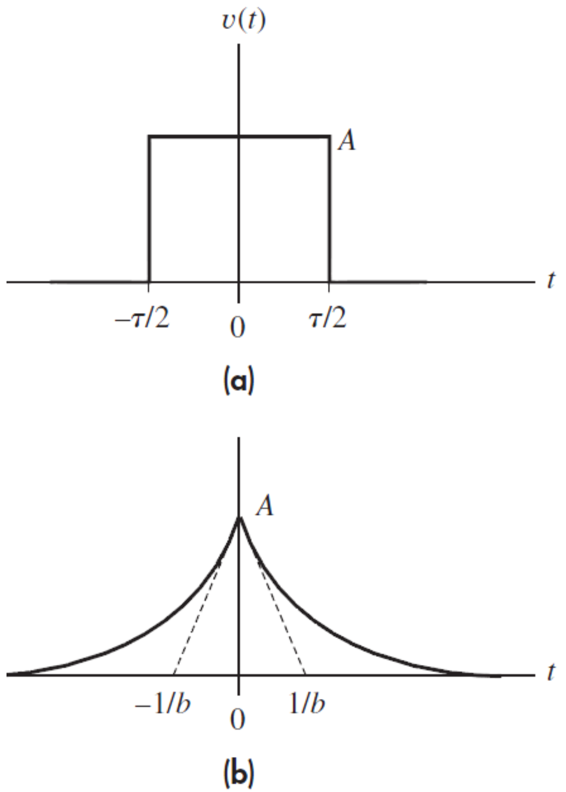
$$P = \sum_{n=-\infty}^{\infty} c_n c_n^* = \sum_{n=-\infty}^{\infty} |c_n|^2$$

Transformada de Fourier



Transformada de Fourier

Definición



$$v(t) = \sum_{n=-\infty}^{\infty} c(nf_0) e^{j2\pi n f_0 t}$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{1}{T_0} \int_{T_0} v(t) e^{-j2\pi n f_0 t}\, dt \right] e^{j2\pi n f_0 t}$$

$$v(t) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} v(t) e^{-j2\pi f t}\, dt \right] e^{j2\pi f t}\, df$$

Transformada de Fourier

Propiedades simples

$$V(0) = \int_{-\infty}^{\infty} v(t)\, dt$$

$$V(-f) = V^*(f)$$

$$V(-f)| = |V(f)|$$

$$\arg V(-f) = -\arg V(f)$$

Transformada de Fourier

Propiedades simples

$$V(0) = \int_{-\infty}^{\infty} v(t)\, dt$$

$$V(-f) = V^*(f)$$

Cuando?

$$|V(-f)| = |V(f)|$$

$$\arg V(-f) = -\arg V(f)$$

Transformada de Fourier

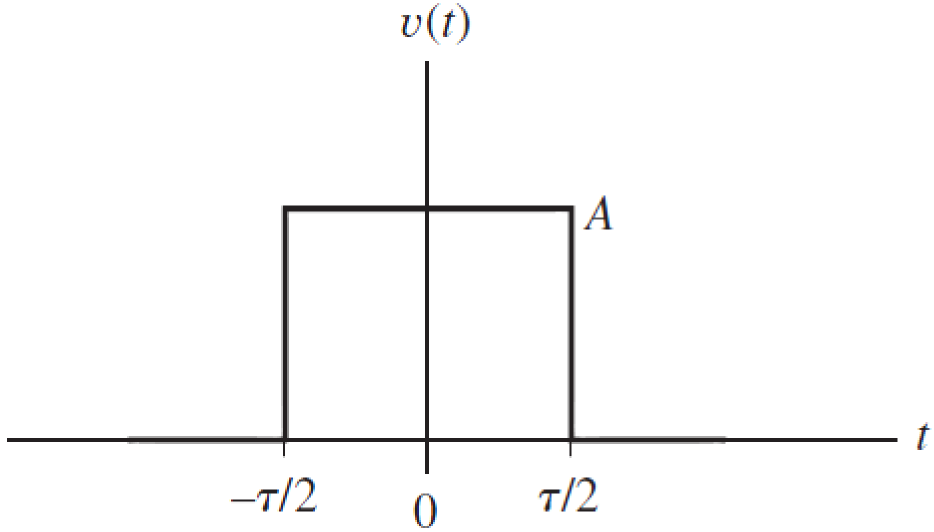
Propiedades simples

Teorema de Rayleigh

$$E = \int_{-\infty}^{\infty} V(f)V^*(f)\, df = \int_{-\infty}^{\infty} |V(f)|^2\, df$$

Transformada de Fourier

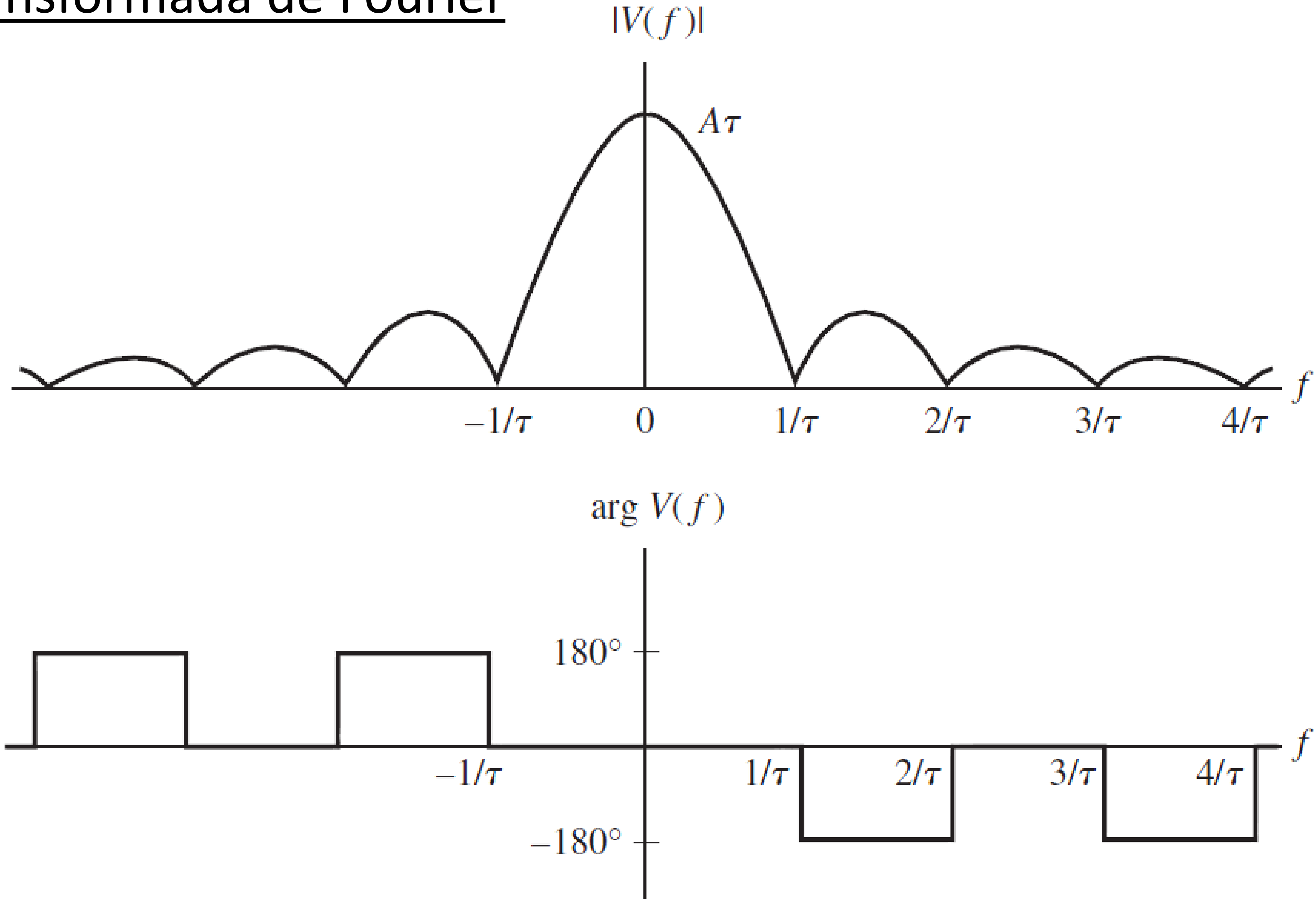
Ejemplo



$$V(f) = \int_{-\tau/2}^{\tau/2} A e^{-j2\pi ft}\, dt = \frac{A}{\pi f} \sin \pi f \tau$$

$$= A\tau \operatorname{sinc} f\tau$$

Transformada de Fourier



Rectangular pulse spectrum $V(f) = A\tau \text{ sinc } f\tau$.

RESUMEN CLASE 6 (y 7)

1 . El espectro de frecuencias es una representación alternativa de la señal temporal, que identifica a esta de manera unívoca
2. Hay varias maneras distintas de representar un espectro, dependiendo de cómo se describa o represente la señal en el tiempo, y se elijan las funciones de la base ortogonal.
3. Las señales periódicas pueden representarse mediante series de Fourier: los espectros resultantes pueden ser unilaterales o bilaterales, según el tipo de funciones elegidas como base. Resultan espectros discretos.
4. La potencia (de una señal de potencia) puede calcularse a partir de las componentes espectrales (Parseval). Para espectros continuos, es calculable la energía (Rayleigh).