Sistemas Lineales

¿Superposición?

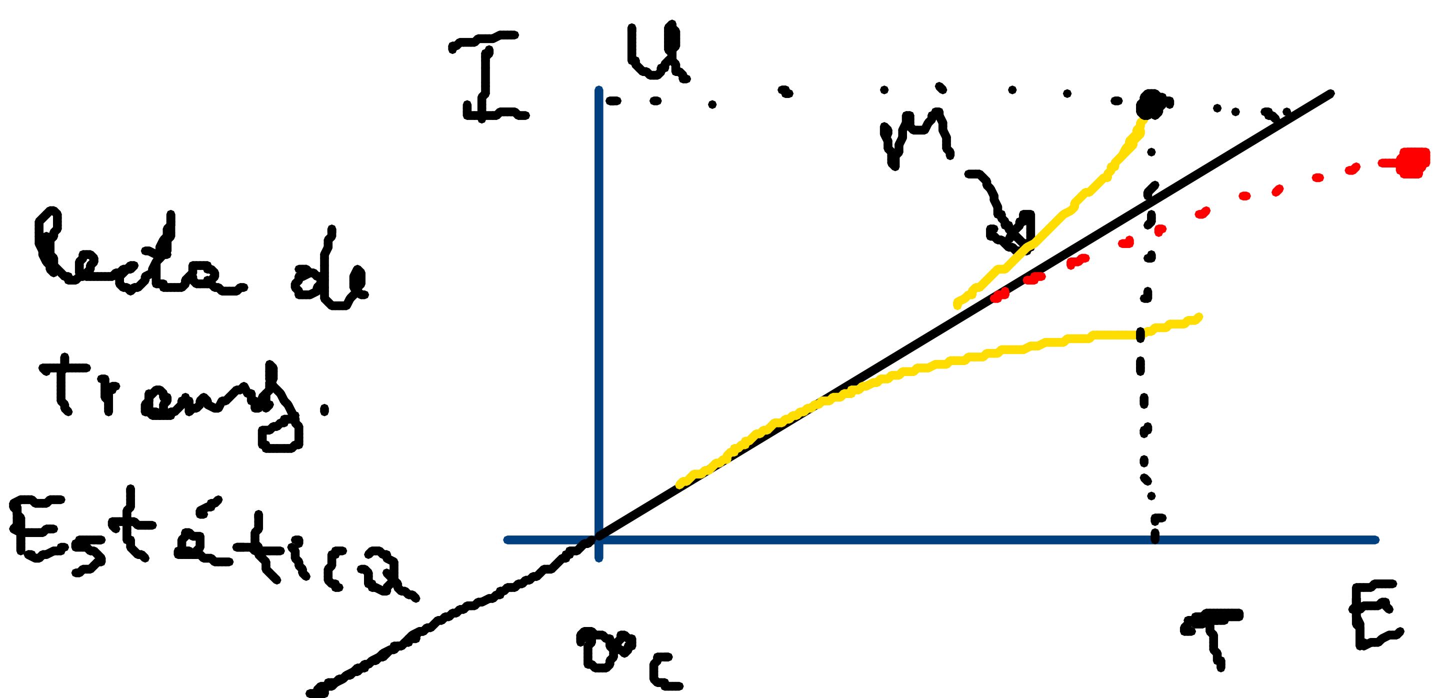
$E_1(t) \rightarrow \boxed{Sis} \rightarrow R_1(t)$

$E_2(t) \rightarrow R_2$

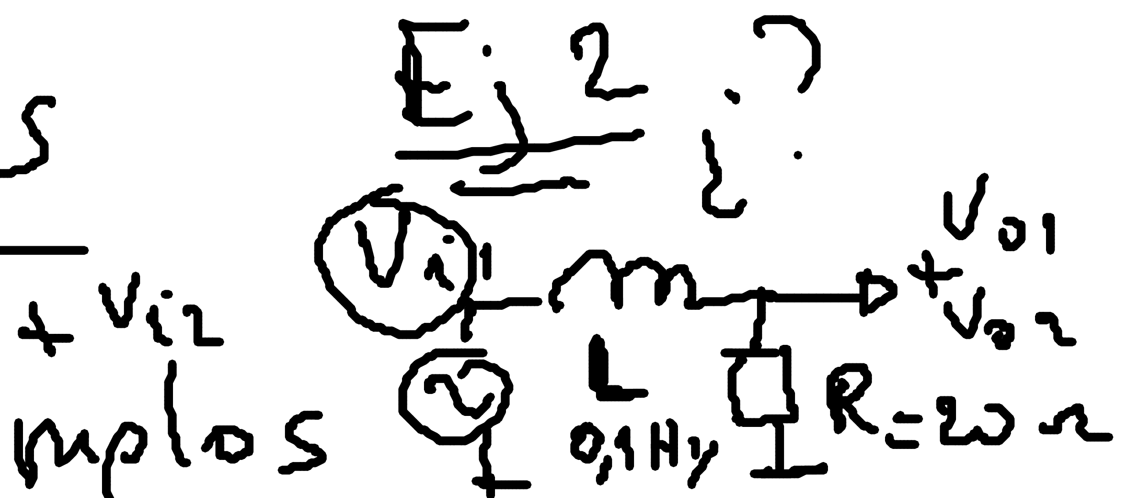
$$\boxed{R(t) = E(t) * h(t)}$$

$$\boxed{E_t = a E_1 + E_2 . b \longrightarrow R_t = a R_1(t) + R_2(t) . b}$$

$$. G . E_1 \longrightarrow R = G . R_1$$



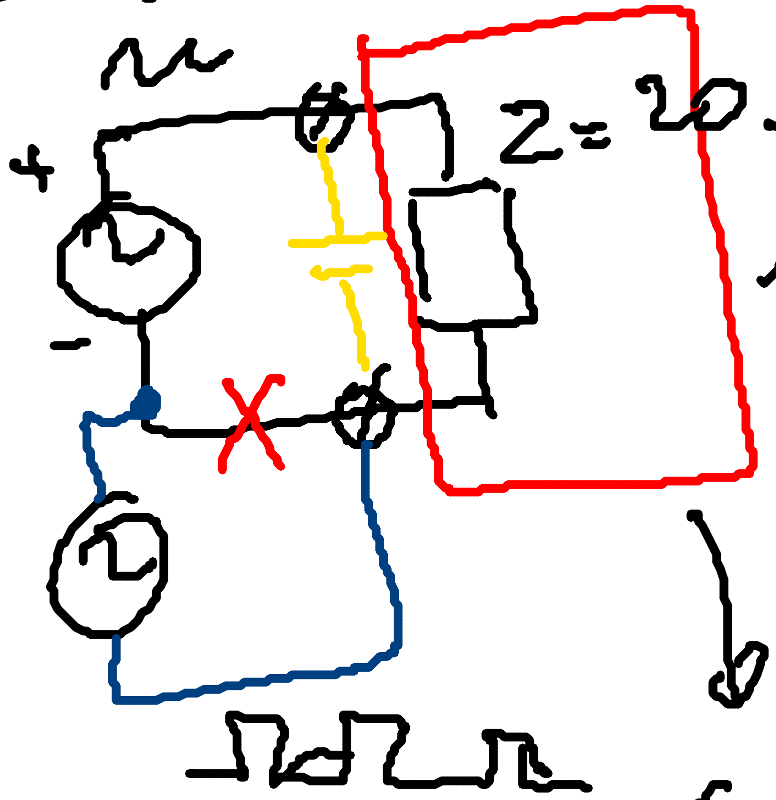
Ej 2 ¿?



n ecuaciones
n incógnitas
única solución

→ 0,99,9

Ejemplos

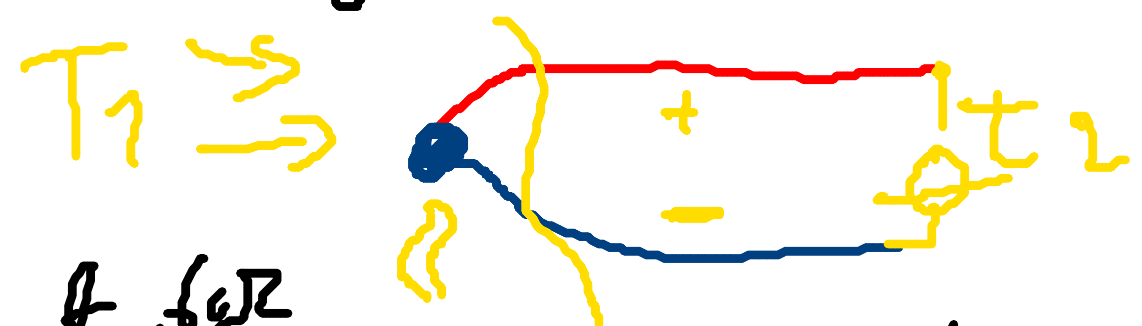


$R_1(t) = i_1(t)$

$R_2(t) = i_2(t)$

$E = I . Z$

Calentador



M = 10 µV/°C
40 mV

$E_1 \rightarrow \boxed{A} \rightarrow R = A . E_1$

Modelos Lineales

"Lindos"

Cómodos

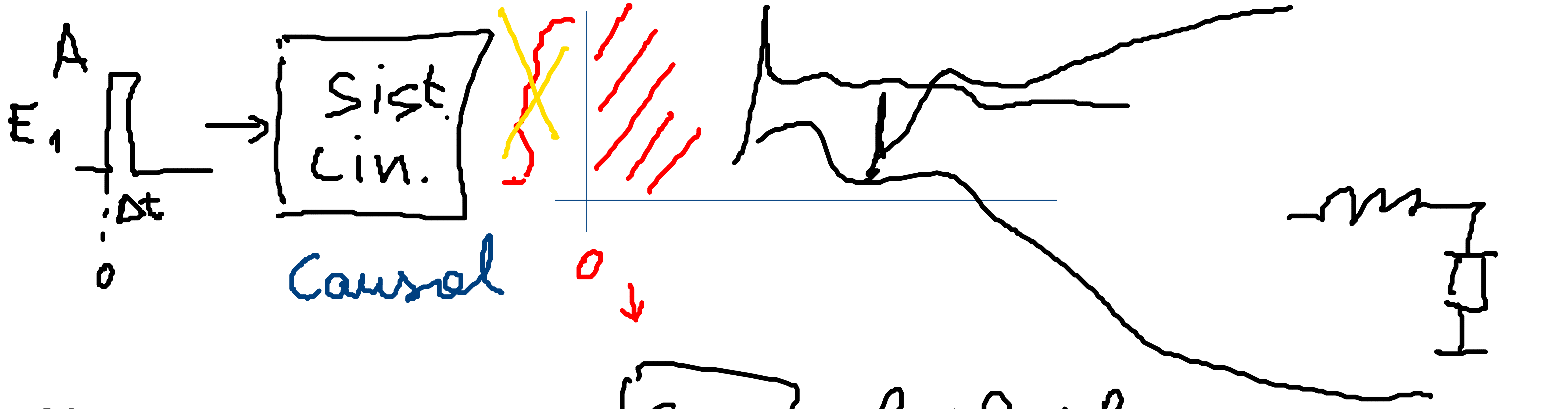
$E = I . Z$ → 20 Ω

→ $Z = f(t)$; $f(s)$

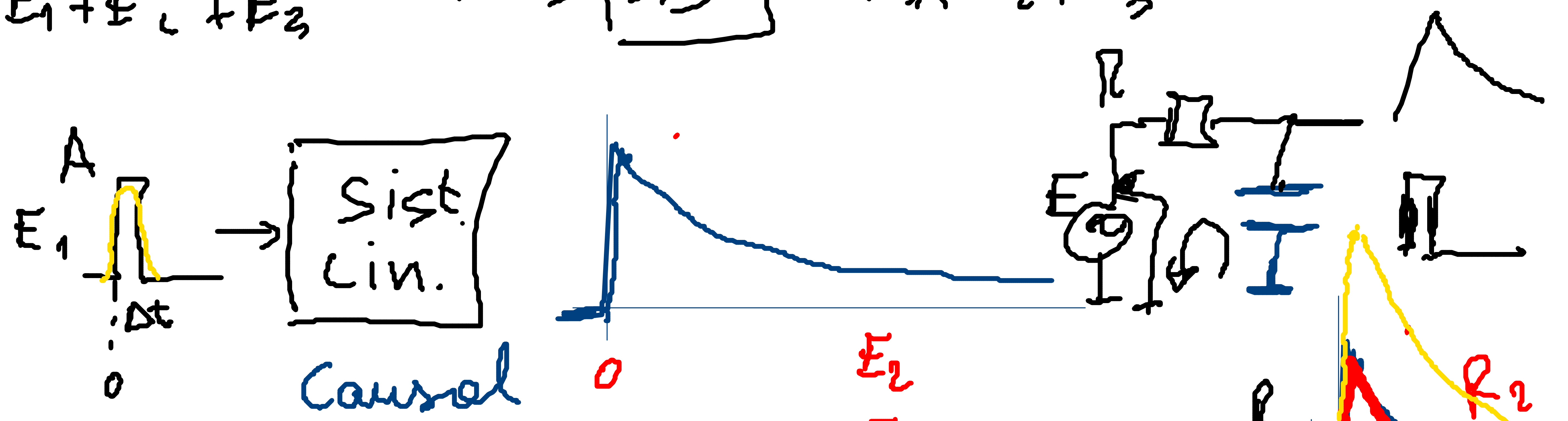
220 VCA   5 ACA → 1 kW

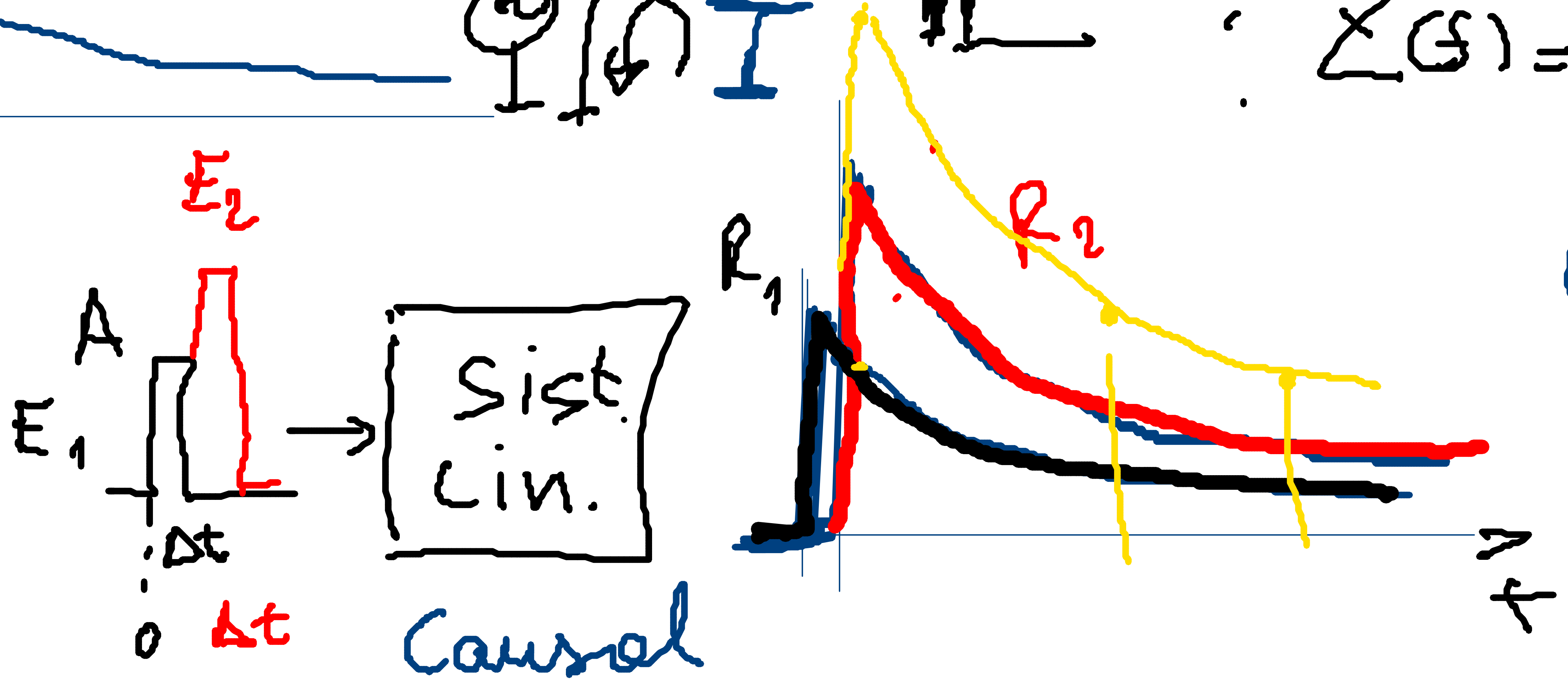
1000 VCA → 2 A → ≈ 4 kW

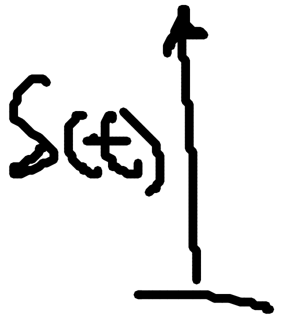
Convolución y Sistemas Lineales



$E_1 + E_2 + E_3 \longrightarrow$ Sis $R_1 + R_2 + R_3 \ldots$







Convol...

Operación mat entre funciones

"Td F" .... $\langle x(t), y(t) \rangle$

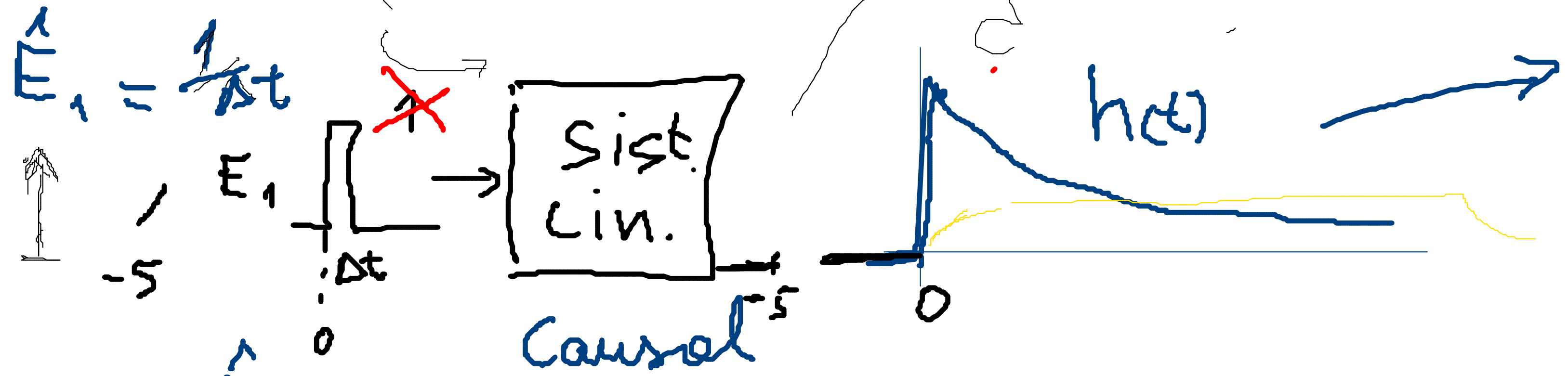
$$c(t) = x(t) * y(t)$$

$$C(f) = X(f) \cdot Y(f)$$

$$Z(t) = g(t) \cdot h(t)$$

$$Z(f) = G(f) * H(f)$$

$$R_E = R_1 + R_2$$



Re TdF de h → F.d.T.

$H(f)$:

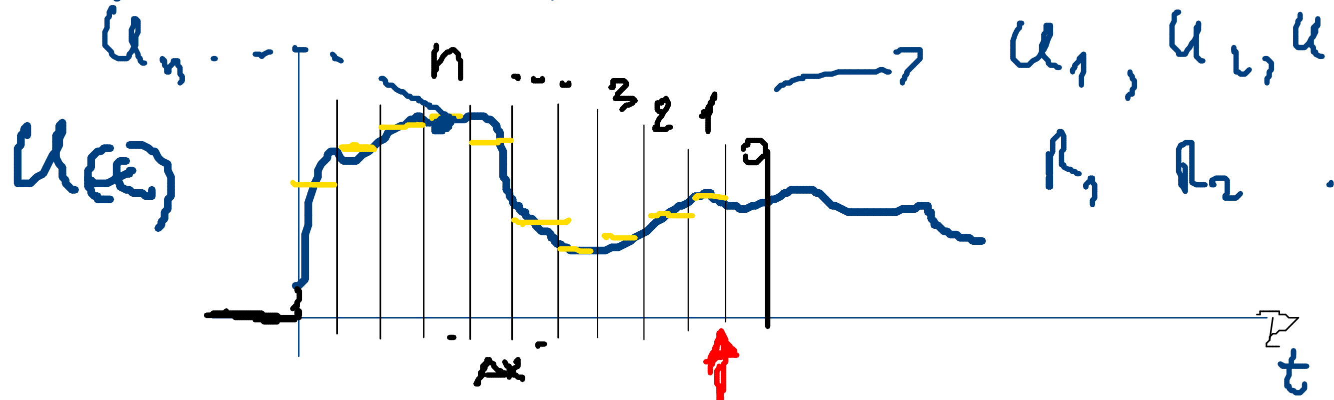
$j2\pi f$

$S = \sigma + j\omega$

TdL unilateral

↓

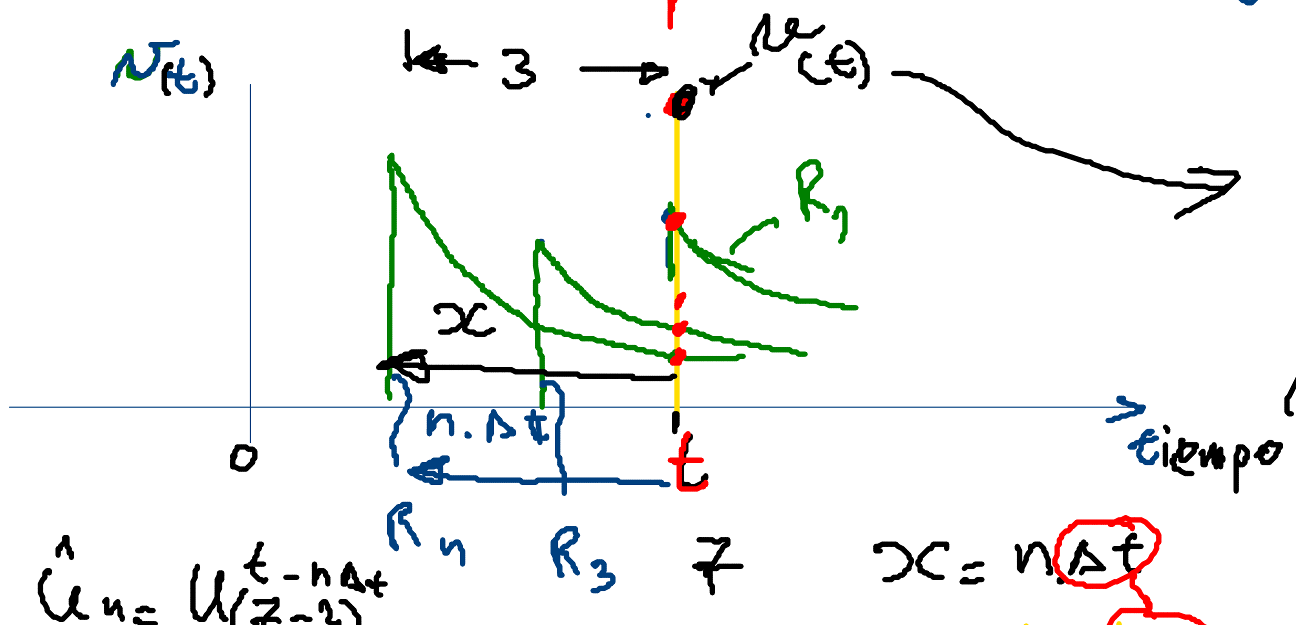
transitorio



$u_1, u_2, u_3, u_4, \dots u_n$

$R_1 \quad R_2 \quad \dots\dots$

$$R_n = \frac{\hat{U}_n}{\hat{E}_1} \cdot h(t) = \hat{u}_n \cdot h_n(t) \cdot \Delta t$$



$$v(t) \simeq \sum_{n=0}^{N} \hat{u}_n \, h_n(t) \cdot \Delta t$$

$$v(t) \simeq \sum_{n=0}^{N} u(t-n\Delta t)\, h(n\Delta t) \cdot \Delta t$$

$\hat{u}_n = u^{t-n\Delta t}_{(7-3)}$

$x = n\Delta t$

$= n.\Delta x$

(t) → u comienza en $t=0$

→ u comenzando desde $t=-\infty$

$$v(t) \simeq \sum_{n=0}^{N} u(t-x) \cdot h(x) \cdot \Delta x = \int_{0}^{\infty} u(t-x) \cdot h(x) \cdot dx$$

$\Delta x \longrightarrow 0 \qquad N \longrightarrow \infty$

0 → h comienza en $t=0$

$-\infty$ → h comienza en $-\infty$

$$v(t) = u(t) * h(t)$$

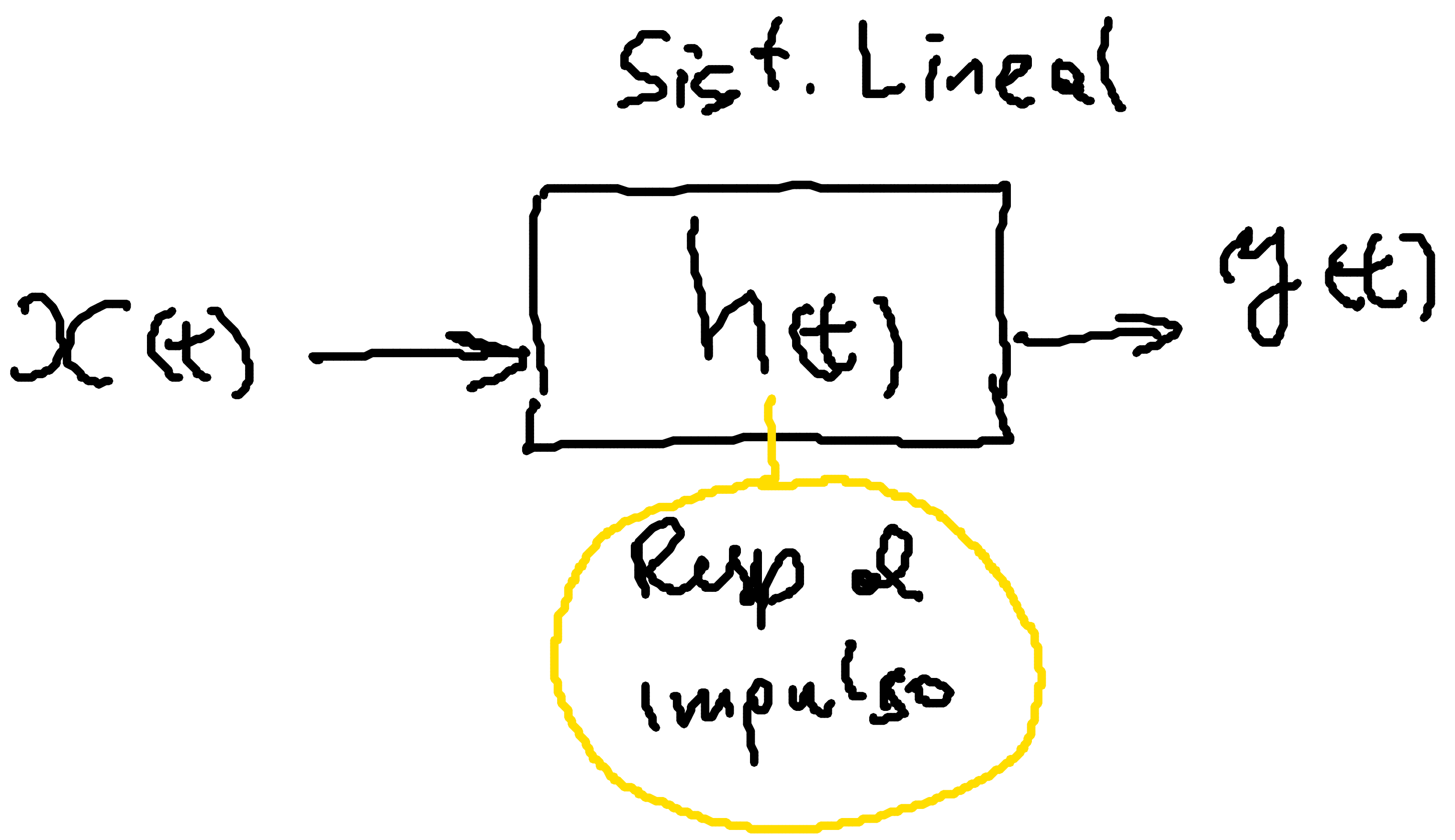
$$v(t) = \int_{-\infty}^{\infty} u(t-x) \cdot h(x)\, dx$$

$\boxed{z = t - x}$

$$v(t) = \int_{-\infty}^{\infty} u(z) \cdot h(t-z) \cdot dz$$

"Principios de Electrotecnia"

Netushil - Strajov p.265. → Principio (Integral de Duhamel

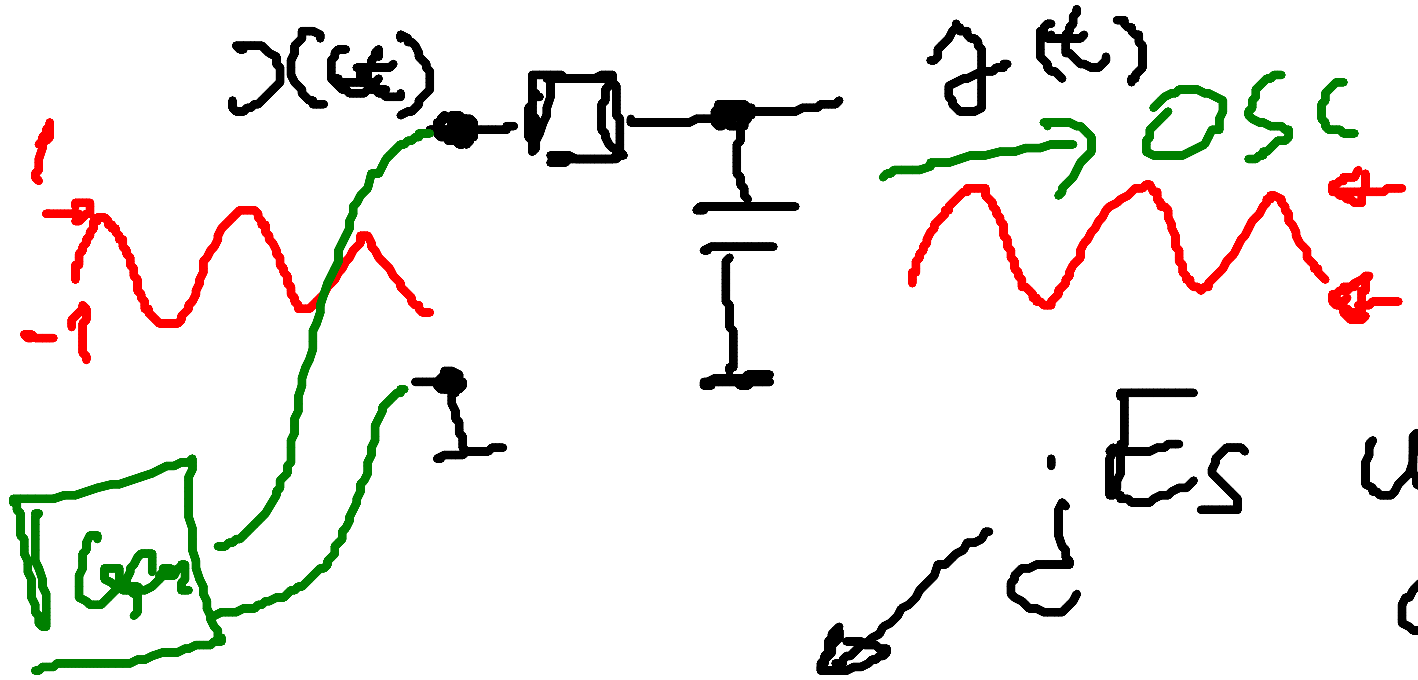


$$y(t) = x(t) * h(t)$$

$$Y(f) = X(f) \cdot H(f)$$

Función de Transferencia

$H(s)$



$$H(f) = \frac{1}{RC} \cdot \frac{1}{j\omega + \frac{1}{RC}} = \frac{1}{jRC\omega + 1}$$

$$H(s) = \frac{1}{sCR + 1}$$

¿Es un espectro?

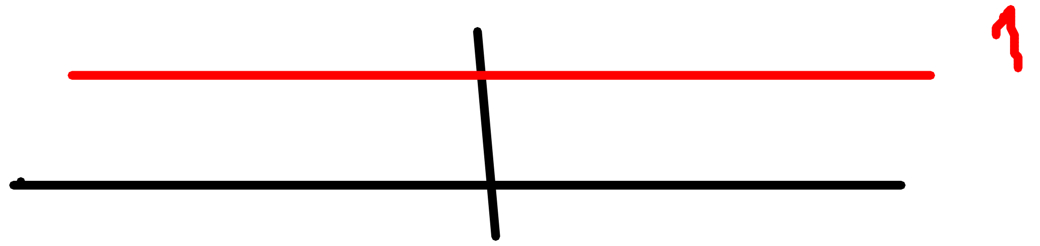
$$Y(f) = H(f) \cdot X(f)$$

¡¡No!!

$$\boxed{H(f) = \frac{Y(f)}{X(f)}}$$

si $x(t) = \delta(t) \rightarrow X(f) = 1$

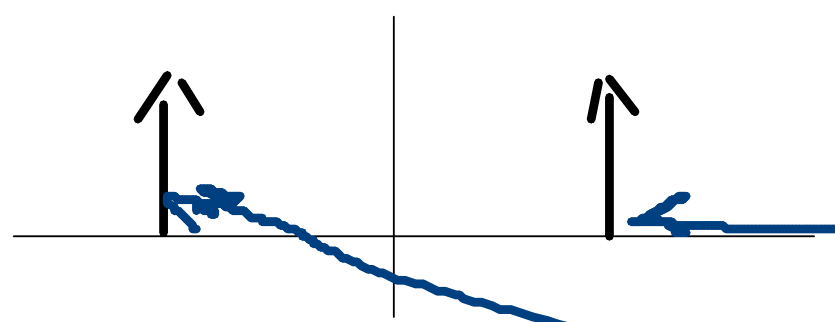
$\Rightarrow Y(f) = H(f) \cdot 1$



¿Respuesta en frecuencia?

$x(t) = \cos \longrightarrow \boxed{H(f)} \longrightarrow y(t)$

$X(f)$

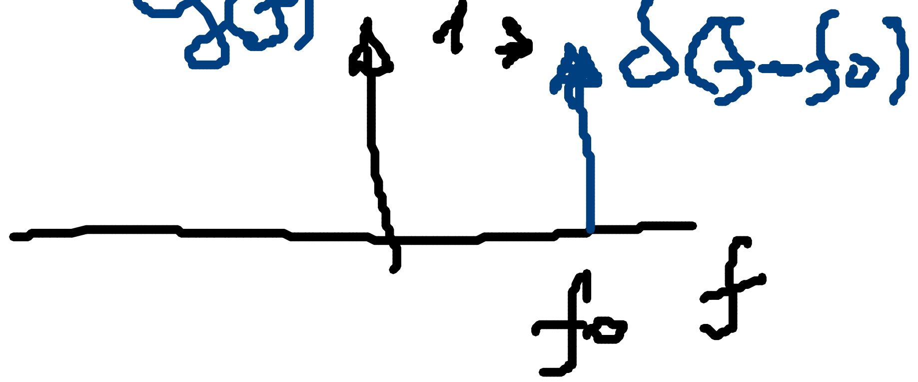


$$\cos(2\pi f_0 t) = \frac{e^{j2\pi f_0 t}}{2} + \frac{e^{-j2\pi f_0 t}}{2}$$

¿$g(t)$?

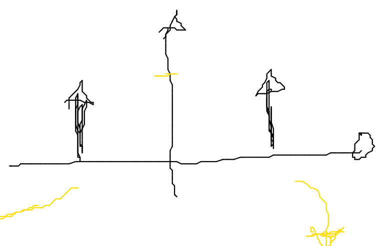
$e^{j f_0 2\pi t}$

$G(f)$



$$H(f) = \frac{1}{j2\pi fRC + 1}$$

$X(f)$



$$Y(f) = H(f) \cdot X(f) \quad ; \quad x(t) = \cos 2\pi f_0 t$$

$$= |H(f)| \angle \phi(f) \cdot X(f)$$

$$= |H(f)| \cdot e^{j\phi(f)} \cdot X(f)$$

"Anti Alias"

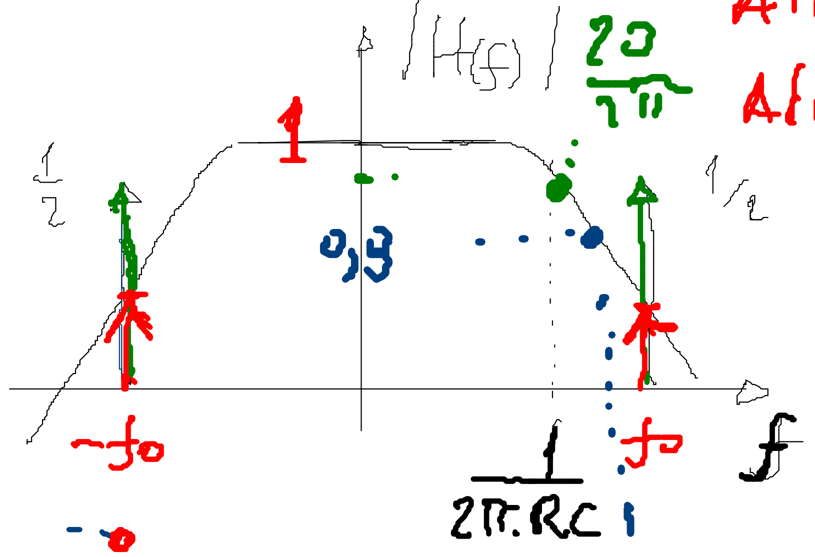
Aliasing

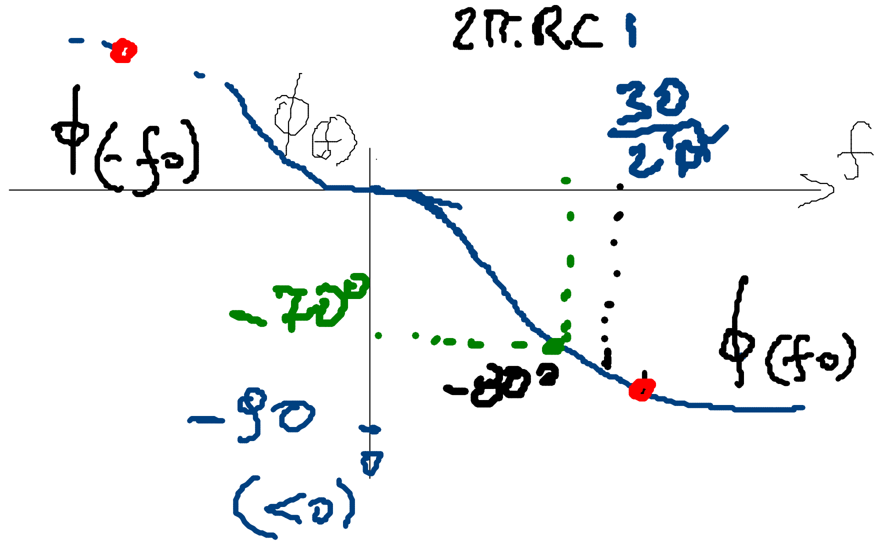
$$Y(f) = \frac{1}{2} |H(f)| e^{j\phi(f)} \cdot \left[\delta(f+f_0) + \delta(f-f_0) \right]$$

$$Y(f) = \frac{1}{2} |H(f_0)| \left[e^{j\phi(-f_0)} \delta(f+f_0) + e^{j\phi(f_0)} \cdot \delta(f-f_0) \right]$$

$$y(t) = |H(f_0)| \left[\frac{e^{j\phi(f_0)} \cdot e^{-j2\pi f_0 t} + e^{j\phi(f_0)} \cdot e^{+j2\pi f_0 t}}{2} \right]$$

$$= |H(f_0)| \left[\frac{e^{-j(2\pi f_0 t + \phi(f_0))} + e^{+j(2\pi f_0 t + \phi(f_0))}}{2} \right]$$

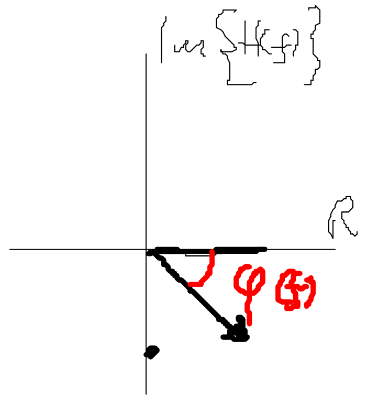




$$\boxed{\begin{aligned} x(t) &= 1 \cdot \cos(2\pi f_0 t) \\ y(t) &= |H(f_0)| \cos[2\pi f_0 t + \phi(f_0)] \end{aligned}}$$

$$H(f) = \frac{1}{j2\pi fRC + 1} = \frac{1 - j2\pi fRC}{4\pi^2 f^2 R^2 C^2 + 1}$$

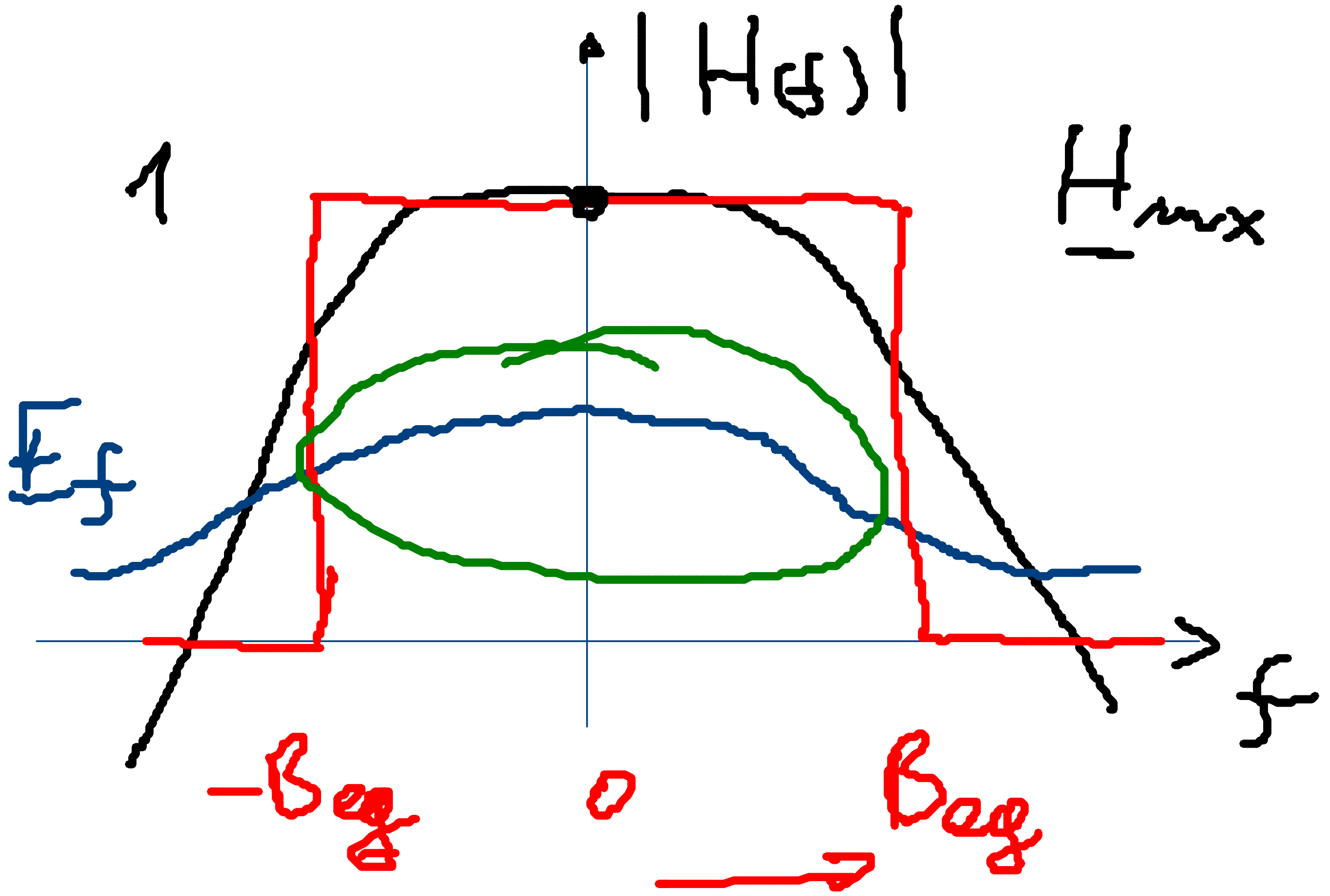
$$\phi(f) = \arctan(-2\pi fRC)$$

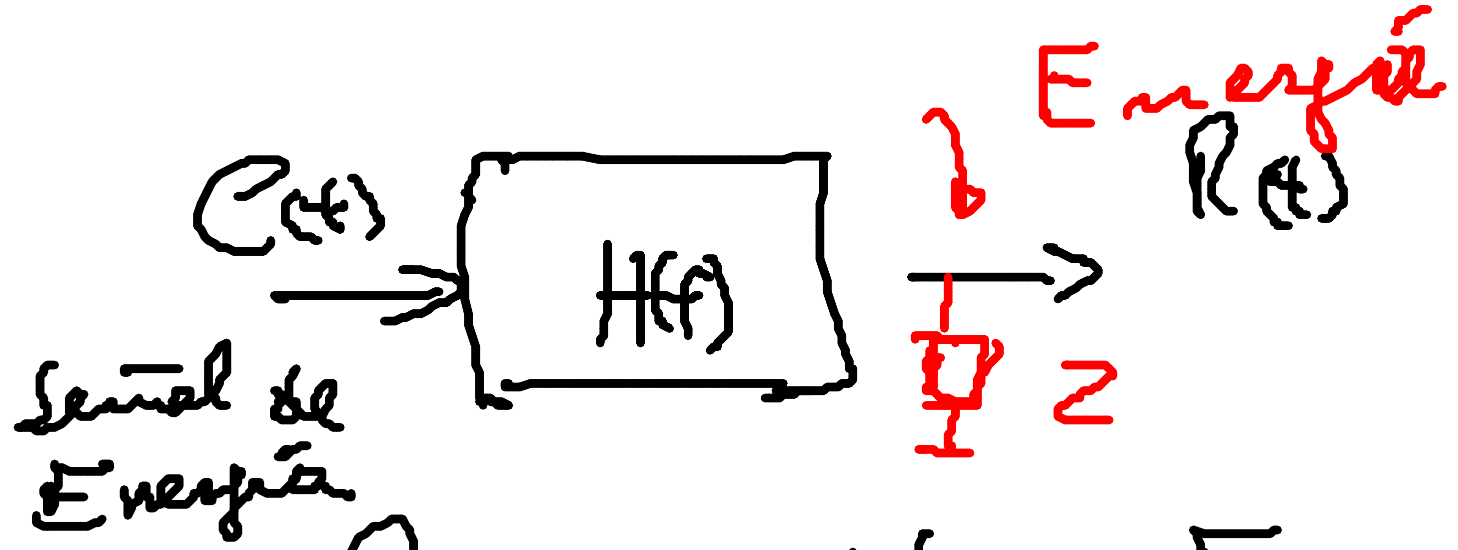


$H(j\omega)$

$$x(t) = 2\cos(30t + 10°) + 5\,\mathrm{sen}(20t - 17°)$$

$$y(t) = 2 \cdot \underbrace{0{,}8}_{\left|H\left(\frac{30}{2\pi}\right)\right|} \cdot \cos(30t - 80° + 10°) + 5 \cdot \underbrace{0{,}9}_{\left|H\left(\frac{20}{2\pi}\right)\right|} \cdot \mathrm{sen}(20t - 17° - 70°)$$





Espectro de Pot

$$S_x(f) = \lim_{T \to \infty} \frac{1}{T} X_T(f)$$

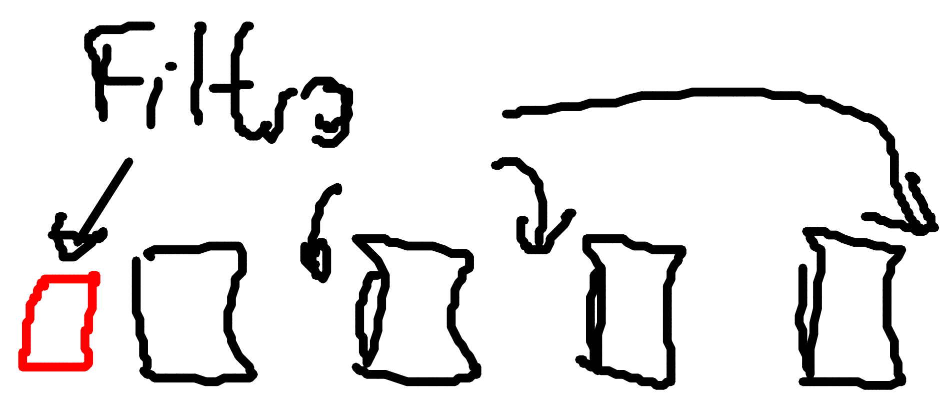
$$R(f) = H(f) \cdot E(f)$$

$$|R(f)|^2 = |H(f)|^2 \cdot |E(f)|^2$$

$$\int_{-\infty}^{\infty} |R(f)|^2 df = \int_{-\infty}^{\infty} \dots\dots$$

Filtros Normalizados

"Ancho de Banda Equivalente"



Energía Salida $= \int_{-\infty}^{\infty} 1 \cdot |H(f)|^2 \cdot df = \int_{-\infty}^{\infty} H_{mx}^2 \cdot df$

$= \cancel{2} \cdot \int_0^{\infty} |H(f)|^2 df = \cancel{2} \cdot B_{eq} \cdot H_m^2$

$$B_{eq} = \frac{\int_0^{\infty} |H(f)|^2 df}{H_m^2}$$