A Matlab-Based Teaching Of The Two-Stub Smith Chart Application For Electromagnetics Class

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Abstract - **This paper specifically presents a *Matlab*-based teaching approach to the two-stub Smith chart problem that is typically covered in an undergraduate course on electromagnetic fields. *Matlab* provides a powerful computation and display platform by means of which fundamental and advanced concepts relating to transmission lines can be understood and implemented. The paper restates the solving techniques of a two-stub Smith chart problem in terms of a process consisting of steps. These steps can be implemented as modules written in *Matlab* scripts. This modular approach using *Matlab* enhances the students' understanding of the Smith chart technique as a solution process with the specific objectives of each module clearly identified. Furthermore, the computations are more accurate and the results can be displayed and interpreted in a more effective manner. The students taking the course completed an on-line survey to rate the effectiveness of the *Matlab* tool in solving the two-stub transmission line problem. The responses to the survey reaffirmed the belief that solutions to problems involving concepts that can be challenging to grasp are learned more effectively when the steps of the solution are associated with processing modules comprising clearly defined input-output relationships.**

Index Terms – Smith chart, *Matlab* application, two-stub transmission line, electromagnetic problem.

INTRODUCTION

The study of the concepts related to transmission lines form an integral part of a typical undergraduate course on electromagnetic fields. These concepts are better presented by incorporating modern tools, such as *Matlab*, for an accurate analysis of the problem and the display of the outcomes. The text on "*Fundamentals of Electromagnetics with Matlab*" [1] is a good example of the use of *Matlab* to illustrate concept and application in the different topics of electromagnetic fields. However, the text provides *Matlab* applications for only single-stub problems and to the best of the authors' knowledge, there are no teaching utilities available to solve two-stub problems. This paper focuses on the approach used to conduct projects for solving a two-stub transmission line problem with the Smith chart using the scripts that were developed by the authors. For instance, a typical two-stub transmission line problem requires students

to compute the shortest length of the required two stubs of transmission lines to achieve no reflection from the load end. Through the use of the traditional paper Smith chart, students tend to focus more on the mechanics of using the Smith chart rather than the understanding of the Smith chart itself. However, with the enhancement on the scripts provided by the text and additional developed scripts, the students were able to walk through the entire process of solving a two-stub transmission line problem with the following emphasis:

- gain better understanding of the construction of a Smith chart;
- better understand the concept of reflection coefficient and its relation to Smith chart;
- understand the choices of possible shortest lengths as solutions with changes in the requirements.

The authors believe students learn better with modular scripts given that require them to figure out what each module does in the process of solving a two-stub problem. The paper reports on the lesson learned and how favorably students appreciate this approach to make difficult subject matter easier to comprehend.

A two-stub transmission line problem is first introduced in the next section and followed by a step-by-step approach of how *Matlab* modular scripts are applied in solving the problem along with the explanation of the developed scripts. The authors will describe also the approach used and the lesson learned.

THE TWO-STUB TRANSMISSION LINE PROBLEM

The problem statement which the students must solve is: A 50Ω coaxial line is terminated in load impedance $Z_L = 100 + j50\Omega$ with two stub tuners at $\lambda/8$ spacing as shown in Figure 1. Find the shortest length for (a) l_1 and (b) l_2 so that there is no reflection at x (VSWR = 1), (c) VSWR on $\lambda/8$ line between the load and stub 1 [2].

Of course, the problem does not restrict itself to this example. But, as an illustration, the process of solving this type of problem is illustrated specifically with this example. The traditional approach of solving this example using a Smith chart is given in Figure 2. The solution involves the following steps:
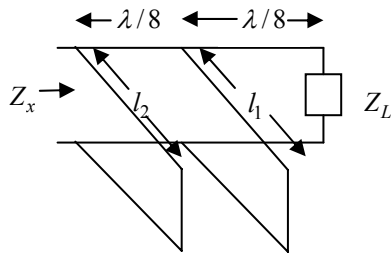
FIGURE 1
A TWO-STUB TRANSMISSION LINE

- step1: locate the normalized load impedance, Z_L;
- step2: locate the corresponding admittance, y_L;
- step3: move y_L $\lambda/8$ away from the load end to y_1;
- step4: draw the rotated unit circle;
- step5: move along the resistance circle of y_1 to the interaction of the rotated unit circle, y_2;
- step6: compute the first shortest length, l_1, based on the susceptance difference between y_1 and y_2;
- step7: move y_2 to y_3 on the intended unit circle by rotating the rotated unit circle $\lambda/8$ clockwise;
- step8: compute the second shortest length, l_2, based on the susceptance of y_3.

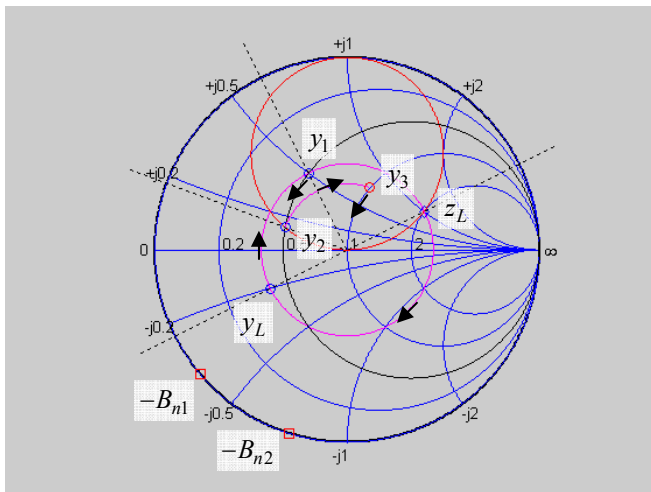Figure 2 shows the key locations on Smith chart for the above steps.

FIGURE 2
A TWO-STUB TRANSMISSION LINE SOLUTION

MATLAB SOLVING APPROACH

This section illustrates using *Matlab* scripts developed to solve the example problem step-by-step. All the scripts

illustrated here are available for download at [3]. Listed below are the modular scripts given to students for solving this example problem.

- **simpleSmithChart** – plot simplified Smith Chart
- **zload** – plot Z_L
- **yL** --- compute and plot the corresponding load admittance
- **zin** --- compute and plot impedance at $x = \lambda/8$
- **rotatedG1Circle** --- draw the rotated $G_n = 1$ circle
- **rCircle** --- draw the resistive circle
- **zOnG1Circle** --- compute the intersection impedance between the resistive circle and the rotated $G_n = 1$ circle
- **plotZ** --- plot impedance on Smith Chart
- **vswrCircle** --- plot VSWR circle
- **lengthOne** --- compute l_1
- **moveZtoG1Circle** --- plot z moving back on the $G_n = 1$ circle
- **lengthTwo** --- compute l_2

Step 0: Use **simpleSmithChart** to first plot the basic Smith Chart. This module was developed based on the example script by the author of the text "Fundamentals of Electromagnetics with *Matlab*" [1]. It essentially plots the layout of a Smith Chart based on a few selected resistive and reactive values using the following two equations:

$$\left(\Gamma_r - \frac{r}{r+1}\right)^2 + \Gamma_i^2 = \left(\frac{1}{r+1}\right)^2 \tag{1}$$

$$\left(\Gamma_r - 1\right)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \tag{2}$$

where, Γ_r and Γ_i are the real and imaginary part of the reflection coefficient, and r and x are the real and imaginary part of the normalized impedance respectively. This set of equations allow students to recognize them as equations for a family of circles in a plane whose axes are labeled as Γ_r and Γ_i. The center and radius of each circle are determined by the value of the normalized resistance r and the normalized reactance x. This module re-enforces the understanding of the inner construction of a Smith chart right from the beginning.

Step 1: Use **zload**(Z_c, Z_L) to locate and plot the normalized load impedance on Smith Chart. This module takes in two parameters: the characteristic impedance of the transmission line, Z_c and the load impedance, Z_L as inputs and gives normalized load impedance and the corresponding VSWR value as output. This module makes use of the fact that the reflection coefficient is a complex quantity that can be expressed in polar coordinates and is related the normalized load impedance in the following manner:

$$\Gamma = |\Gamma| e^{j\theta_L} = \frac{z_L - 1}{z_L + 1} \qquad (3)$$

where z_L is the normalized load impedance. Since the load impedance can be complex, there will be a phase angle θ_L associated with the reflection coefficient. Equation (3) is primarily used throughout the rest of the modular scripts in locating and plotting the impedance on Smith chart. However, the corresponding VSWR is computed based on its relation the reflection coefficient as follows:

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} \qquad (4)$$

In this example, the z_L turns out to be $2 + j1\Omega$ and the corresponding VSWR is 2.44. The advantage of using script for computing VSWR instead of the customary practice of reading off the Smith chart is the re-enforcement of how VSWR is computed and the answer is more accurate than round off error in reading the chart.

Step 2: Use **yL(** z_L **)** to locate the corresponding normalized admittance, y_L. This module takes normalized load impedance computed from Step 1 as input parameter and gives the normalized admittance as output. It plots the admittance on Smith chart using (3). In this case, y_L turns out to be $0.4 - j0.2\Omega$.

Step 3: Move y_L $\lambda/8$ away from the load end to y_1 by using **zin(** λ_x, Z_c, z_L **)** where λ_x is the distance away from the load in λ, Z_c the characteristic impedance and z_L the normalized load impedance. In this example, λ_x is $1/8$. **zin** essentially computes impedance at any λ_x away from the load using the following equation:

$$z_{in} = \frac{z_L + j \tan(k\lambda_x)}{1 + j z_L \tan(k\lambda_x)} \qquad (5)$$

where $k = 2\pi$. In addition, **zin** plots the corresponding reflection circle on the Smith chart by making use of the following relation:

$$z_{in} = \frac{1 + |\Gamma| e^{j\phi}}{1 - |\Gamma| e^{j\phi}} \qquad (6)$$

where,

$$\phi = \theta_L - 2k\lambda_x \qquad (7)$$

Since the magnitude of the reflection coefficient remains a constant as it moves along the transmission line back toward the signal generator. The phase shift ϕ is the only thing that changes and is linearly proportional to the distance λ_x. As a result, the reflection circle can be plotted by rotating the initial value of the load impedance along a circle whose radius is equal to the reflection coefficient $|\Gamma|$ and the amount of rotation depends on the distance $2k\lambda_x$ where this phase angle must be subtracted from the initial value used to locate the load impedance on the Smith chart. In this example problem, y_1 is computed to be $0.5 + j0.5\Omega$.

Step 4: Use **rotatedG1Circle(** x, y **)** to draw the rotated unit circle. This module accepts x and y as the coordinates for the center of the rotated unit circle. In this example, the rotated unit circle is 90^0 counterclockwise; hence, the center is $(0, 0.5)$ with respect to the center of the Smith chart. The module could have been written to accept λ_x as input parameter. However, the intention here is to allow students to realize that the rotated circle is a result of shifting the center by using the following circle relation:

$$(\Gamma_r - x)^2 + (\Gamma_i - y)^2 = r^2 \qquad (8)$$

where (x, y) is the center of the circle and r the radius. In this example, $r = 0.5$.

Step 5: Move along the resistance circle of y_1 to the interaction of the rotated unit circle. To accomplish Step 5, use **rCircle(** r **)** to first draw the resistance circle if it has not been drawn by the **simpleSmithChart** by entering the specific r value to **rCircle** module. **rCircle** uses (1) to draw the corresponding resistance circle on Smith chart. Then, use **zOnG1Circle(** r **)** to compute the possible intersection solutions between the resistance circle and the rotated unit circle.

To compute the intersection, (1) and (8) are solved simultaneously for Γ_r and Γ_i. There would be two possible solutions. Substitute each solution into the following equation to obtain the corresponding r and x values.

$$z = \frac{1 + \Gamma}{1 - \Gamma} \qquad (9)$$

or,

$$r + jx = \frac{1 + \Gamma_r + j\Gamma_i}{1 - \Gamma_r - j\Gamma_i} \qquad (10)$$

There are two sets of r and x values as they should be as there are two intersection points between the resistance circle and the rotated unit circle. Students are to determine which solution is the proper solution to use further in the following steps to archive minimum length solutions for

l_1 and l_2, respectively. In this example, the two solutions are $0.5 + j1.866\Omega$ and $0.5 + j0.134\Omega$. The $0.5 + j0.134\Omega$ is the proper solution for y_2.

Step 6: Use **lengthOne(y_1, y_2)** to compute the first shortest length, l_1, based on the susceptance difference between y_1 and y_2. The following equations are used:

$$B_{n,diff} = B_{n2} - B_{n1} \tag{11}$$

$$z_x = jZ_c \tan(k\lambda_x) \tag{12}$$

Equation (12) is used to compute impedance at any point away from a short-circuited load ($Z_L = 0$), in this case, the short-circuited stub line. In this application, $z_x = -B_{diff}$ in (12) and solve for λ_x as the shortest length, l_1. The script **lengthOne** gives $l_1 = 0.1942\lambda$ as the answer and plots $-B_{n,diff}$ on the Smith chart.

Step 7: Use **moveZtoG1Circle(λ_x, y_2)** to move y_2 to y_3 on the intended $G_n = 1$ circle by rotating the rotated $G_n = 1$ circle $\lambda/8$ clockwise. The **moveZtoG1Circle** takes λ_x distance to be rotated back and z_2 the impedance to be moved back to the unit circle. In this example, $\lambda_x = 1/8$ and z_2 is the solution from Step 5. It uses (5) to compute z_3 with z_2 as the starting point. It gives $y_3 = 1 + j0.7321\Omega$ as the answer on the $G_n = 1$ circle. Hence, stub 2 requires to present a susceptance of $-j0.7321\Omega$ in order to take y_3 to the center of the chart where $y_n = 1 + j0\Omega$. When $G_n = 1$, $R_n = 1$ and the impedance presented to the main line at stub 2 is equal to $R_n \times Z_c = 1 \times 50 = 50\Omega$ for a match.

Step 8: Use **lengthTwo(y_3)** to compute the second shortest length, l_2, based on the susceptance of y_3. The script **lengthTwo** is a similar module as **lengthOne** and uses (12) to compute the λ_x, but it takes y_3 only as the only parameter. In this case, $l_2 = 0.1494\lambda$ is the answer. It also plots $-B_{n,3}$ on Smith chart.

The 8 steps described above conclude the use of *Matlab* modular scripts in solving the two-stub transmission line problem.

STUDENT PROJECT

The two-stub transmission line project was given to students, after the concept and principles on transmission line and the application of Smith chart were discussed. Homework was given to solve the single-stub as well as the two-stub

problem using the traditional paper Smith chart. Some homework problems focus on solving simple transmission line problems using *Matlab* as a tool. The equations presented here and the functionality of the modular scripts were discussed in the class. Some simple problems such as (1) locate normalized impedance and its corresponding admittance on Smith chart, (2) compute and plot z_{in} at any point λ_x away from the load illustrated using *Matlab* were given. Towards the end of the semester, this two-stub problem was given as a project. The broad objective was to re-enforce the concepts used to solve the transmission line problem using the Smith chart with the aid of *Matlab*. Specifically, the intent was to improve the understanding of (1) VSWR computation, (2) z_{in} computation at any point λ_x, (3) the technique to solve the two-stub problem, and (4) the use of *Matlab* tools.

Figure 3 illustrates a typical m-file solution to the example problem. The dotted command line indicates that the answer generated at that line is needed for the following command line. For example, yL=yL(ZL) will generate a value for yL and is needed as input for the following command line: y1=zin(1/8,Zc,yL).

```
% steps to solve a two-stub transmission line problem
% step 1: draw Smith chart
simpleSmithChart;
%step 2: define load
Zc = 50;
ZL = 100+j*50;
% step 3: compute admittance
• yL = yL(ZL);
%step 4: compute admittance at 1/8 lambda
• y1 = zin(1/8,Zc,yL);
%step 5: draw rotated circle
rotatedG1Circle(0.0,0.5);
%step 6: draw the r circle
rCircle(0.5);
%step 7: compute the intersection admittance
z=zOnG1Circle(0.5);
%step 8: define y2 and compute the first min length
• y2=0.5+j*0.1340;
plotZ(y2);
vswrCircle(y2);
l1=lengthOne(y2,y1);
%move point back to G1 circle and compute length 2
• y3=moveZtoG1Circle(1/8,y2);
l2 = lengthTwo(-0.7321);
```

FIGURE 3
MATLAB M-FILE SOLUTION

STUDENT FEEDBACK

Each student completed an on-line survey at the end of the semester. The survey was taken by the six students taking the course. The survey consisted of two sections (a) quantitative (b) qualitative. The quantitative section comprised graded responses (on a scale from 0 to 5) to items in the broad category titled '*Effectiveness of Matlab tools in solving transmission line problem*'.

(a) Quantitative section

The items, displayed in Table 1, relate to the following specific question.

Would the use of the Matlab tool aid you in understanding better the following items?

Table 1: Effectiveness of *Matlab* tool

Questions	A	B	C	D	E
Smith chart is formed by two sets of circles	40.0	40.0	20.0	0.0	0.0
compute and plot Zin (input impedance) at any point on the transmission line away from the load	40.0	40.0	20.0	0.0	0.0
VSWR computation	20.0	80.0	0.0	0.0	0.0
identify steps to be taken in order to solve the two-stub transmission line problem	60.0	20.0	20.0	0.0	0.0
better understand the concept of reflection coefficient and its relation to Smith chart	0.0	40.0	40.0	20.0	0.0
Total Class Response:	**32**	**44**	**20**	**4**	**0**

A: Strongly Agree, B: Agree, C: Neutral, D: Disagree, E: Strongly Disagree

Table 1 summarizes the graded response to each item. Most students either agreed or strongly agreed that the use of *Matlab* helped them (a) understand that the Smith chart is formed by two sets of circles, (b) compute and plot the input impedance at any point on the transmission line away from the load, (c) the VSWR computation, (d) improve the identification of the steps to be taken to solve the two-stub transmission line problem. However, the students did not strongly agree that they had a better understanding of the concept of reflection coefficient and its relation to the Smith chart. This was the first time that the *Matlab* tool was being used to re-enforce concepts related to transmission lines. The instructor was developing each module and did not have available all the modules described above when the project was given. Consequently, the students had the additional task to write part of the modules before the implementation. This was quite challenging for the students who had a limited knowledge of the features of the *Matlab* tool. All the modules are now written and functional. In future, the students will be provided these modules and the solution process will be better streamlined.

(b) Qualitative section

The qualitative section asked each student to comment on the following two questions.

(1) *How would you improve the project?*

Most students felt that they required additional preparation in *Matlab* prior to solving the two-stub transmission line problem with the aid of this tool. The students taking this course have employed *Matlab* in some of the other courses they have previously taken, however a review session was recommended. The students who did not have the necessary background in *Matlab* took longer to grasp the modules. Again, this is partly due to the fact that students were required to write part of the scripts initially. If all the modules were available ahead of time, the anxiety felt by students to write scripts in addition to learning the concept would be much reduced if not eliminated.

(2) *Are there any components of the Matlab approach which must be altered?*

One-third of the class did not feel the need to alter the *Matlab* approach. Some students expressed the need for a laboratory session to resolve their doubts and gain more experience programming with *Matlab*.

CONCLUSIONS

The use of *Matlab* as a teaching aid to improve the understanding of concepts related to transmission lines was successful. The association of the conceptual steps with processing *Matlab* modules provides the students with a system-level approach to problem solving. This approach allows the student to recognize the critical input and output relationships among the variables of the problem statement. The students are able to relate to a clearer path leading to the eventual solution. This path consists of intermediate stages which are better understood and integrated into the learning process.

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