TRANSFORMACIÓN DE LAPLACE

Definición

$$\mathcal{L}[f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t)\,dt$$

Propiedades

1. Linealidad

$$\mathcal{L}[a\,f(t) + b\,g(t)] = a\,\mathcal{L}[f(t)] + b\,\mathcal{L}[g(t)]$$

2. Tranformación de funciones trasladadas

$$\mathcal{L}[f(t-L)] = e^{-Ls}\,\mathcal{L}[f(t)]$$

3. Tranformación de la diferenciación real

$$\mathcal{L}\left[\frac{df(t)}{dt}\right] = s\,F(s) - f(0)$$

$$\mathcal{L}\left[\frac{d^2f(t)}{dt^2}\right] = s^2\,F(s) - s\,f(0) - \left.\frac{df}{dt}\right|_{t=0}$$

4. Transformación de la integral temporal

$$\mathcal{L}\left[\int_0^t f(\tau)\,d\tau\right] = \frac{F(s)}{s}$$

5. Teorema del valor inicial

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty}[sF(s)]$$

6. Teorema del valor final

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0}[sF(s)]$$

f(t)	F(s)
$\delta(t)$ (Impulso unitario)	1
1 (escalón unitario)	$\frac{1}{s}$
t (rampa unitaria)	$\frac{1}{s^2}$
t^{n-1}	$\frac{(n-1)!}{s^n}$
e^{-at}	$\frac{1}{s+a}$
$\frac{1}{\tau}e^{-t/\tau}$	$\frac{1}{\tau s+1}$
$\frac{1}{\tau^n(n-1)!}t^{n-1}e^{-t/\tau}$	$\frac{1}{(\tau s+1)^n}$
$\frac{1}{\tau_1-\tau_2}(e^{-t/\tau_1}-e^{-t/\tau_2})$	$\frac{1}{(\tau_1 s+1)(\tau_2 s+1)}$
$1-\frac{\tau_1-\tau_3}{\tau_1-\tau_2}e^{-t/\tau_1}+\frac{\tau_2-\tau_3}{\tau_1-\tau_2}e^{-t/\tau_2}$	$\frac{\tau_3 s+1}{s(\tau_1 s+1)(\tau_2 s+1)}$
$1-e^{-t/\tau}$	$\frac{1}{s(\tau s+1)}$
$1+\frac{1}{\tau_1-\tau_2}(\tau_2 e^{-t/\tau_2}-\tau_1 e^{-t/\tau_1})$	$\frac{1}{s(\tau_1 s+1)(\tau_2 s+1)}$
$1-\frac{(\tau+t)}{\tau}e^{-t/\tau}$	$\frac{1}{s(\tau s+1)^2}$
$\mathrm{sen}(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\,\mathrm{sen}(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{\omega_n}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\,\mathrm{sen}(\omega_n\sqrt{1-\xi^2}\,t)$	$\frac{1}{\frac{1}{\omega_n^2}s^2+\frac{2\xi}{\omega_n}s+1}$
$1-\frac{1}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\,sen(\omega_n\sqrt{1-\xi^2}\,t-\phi)$ $tg\phi=\frac{\sqrt{1-\xi^2}}{-\xi}$ $\quad 0\le\xi<1$	$\frac{1}{s(\frac{1}{\omega_n^2}s^2+\frac{2\xi}{\omega_n}s+1)}$
$\tau(e^{-t/\tau}+\frac{t}{\tau}-1)$	$\frac{1}{s^2(\tau s+1)}$
$\frac{\omega\tau}{\tau^2\omega^2+1}e^{-t/\tau}+\frac{1}{\sqrt{\tau^2\omega^2+1}}\mathrm{sen}(\omega t+\phi)$ $tg\phi=-\tau\omega$	$\frac{\omega}{(\tau s+1)(s^2+\omega^2)}$