Continuous Optimization in Aerospace Structures

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1 INTRODUCTION

Designers intuitively make decisions regarding the choice of material, the shape of the design space, and the distribution of the material in it in order to realize efficient and robust designs that are light and strong as required in the aerospace industry. This experience-based approach has worked well over the decades. But it is not satisfactory anymore for advanced solutions in demanding applications; human intuition might simply miss out on interesting, better designs. Engineering design optimization aims to overcome this by efficiently searching the design space in an automatic and systematic manner for optimal design solutions employing various mathematical, engineering, and computational tools in one framework.

Structural engineering design optimization aims to minimize a measure or a combination of measures of a structure's performance, such as the weight, stiffness, compliance, frequency, for a given set of design variables. Different parameters can be chosen as design variables, such as geometrical properties, material properties, or loadings. Such variables are mathematically adjusted during the automatic design process and are constrained to satisfy physical and manufacturing requirements.

A general simulation-based optimal design process is built around the synthesis of several disciplines as described in the three-column concept by Braibant and Fleury (1986) (see Figure 1). This process consists in the choice of (i) a (design) *optimization model* (ii) an *analysis model*, and (iii) the *optimization algorithms*.

The *optimization model* represents the link between the structural model and the optimization algorithms. This enables the transformation of the design optimization problem into a parametric optimization problem. Such link is realized through the design parameterization process, which involves mainly two steps: the geometric modelling and the design variable definition. First, one builds a geometric model with the definition of all of its dimensions. Then, one identifies the design variables as a subset of the geometric/physical parameters that characterize the structure. The *analysis model* refers to the mathematical determination of the physical behavior of the structure. It represents the structure in terms of a system of ordinary and/or partial differential equations with unknown structural responses such as displacement, velocity, and stress fields. Numerical methods are usually used to obtain approximate solutions to a given accuracy, such as the finite element method (FEM). The *optimization algorithms* denote the suite of numerical procedures that solve the actual optimization problem, such as mathematical programming or discrete optimality criteria methods (see Review of Optimization Techniques). The algorithms can be classified in various

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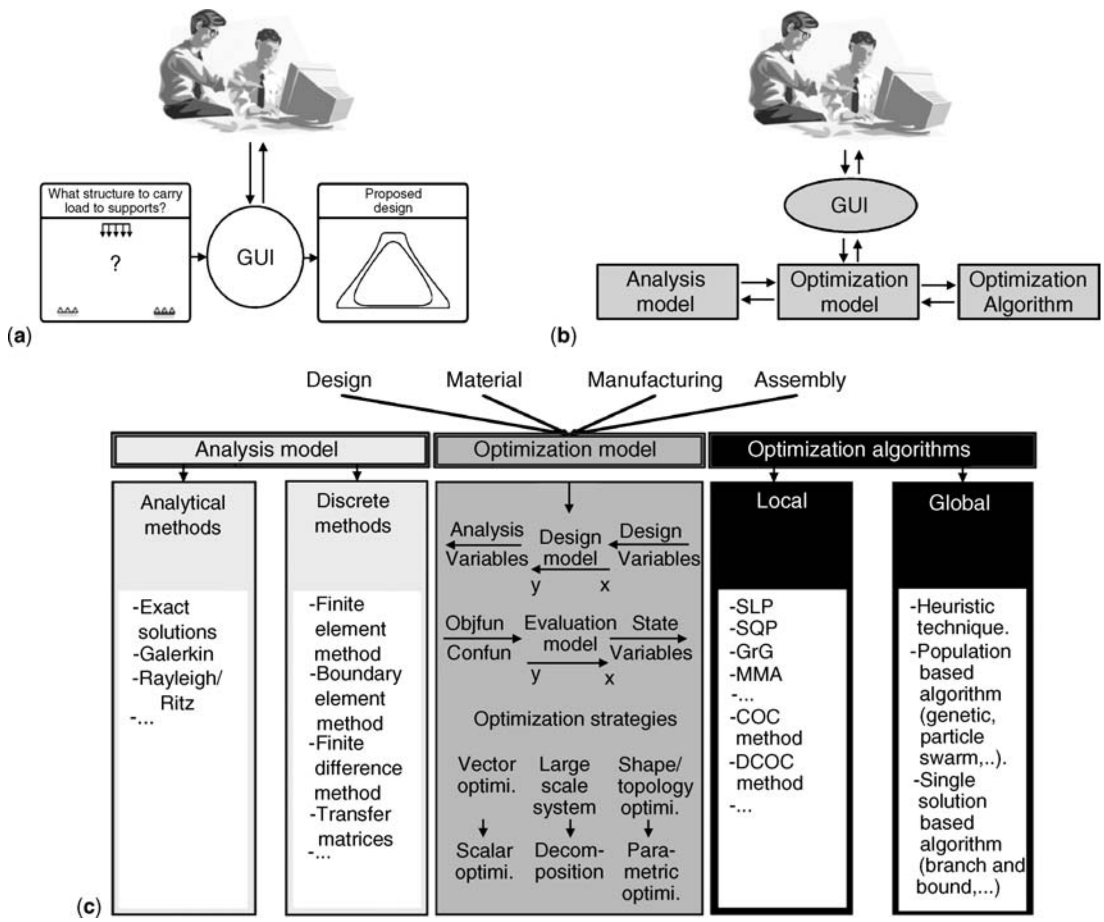

Figure 1. Simulation-based optimal design process: *The three-column concept:* (a) Design problem: given the loads and the supports, what is the “best” structure?; (b) three-column concept; (c) detailed aspects.

manners, such as local/global, gradient-based/gradient-free or deterministic/probabilistic-stochastic methods.

Further aspects are the sensitivity analysis (see Sensitivity Analysis) for gradient-based optimization (see Review of Optimization Techniques) and an interactive graphical user interface. In Hörnlein and Schittkowski (1993), some early implementations of the three-column concept or variations of it in computer codes such as CARAT, SAPOP, CAOS, and OASIS, are described. Later, Sienz and Hinton (1997) added accurate FEM incorporating error estimation (see Error Estimation and Quality Control) and adaptive mesh generation (see Adaptive Mesh Generation and Visualization). Some aspects can also be found in Keane and Nair (2005).

The objective of this section is to illustrate a practical realization of an optimization problem in structural engineering design, which includes the selection of the design variables, the mathematical formulation of the problem, the parameterization and finally the optimization algorithms available for the resolution of that specific type of problem. With that aim, different optimization techniques including topology, size, and shape optimization are carefully formulated in their continuous and discrete approaches. Advanced technologies usually involved in the resolution of practical problems are also briefly reviewed, such as modelling, analysis by the FEM, mesh generation, error estimation, and adaptivity. Finally, several illustrative examples concerning structural engineering design optimization in the aerospace industry are given.

The reader is referred to Computational Optimization for details on computational optimization.

2 TOPOLOGY, SHAPE, AND SIZE OPTIMIZATION

Structural optimization problems are generally classified as:

- **Topology optimization**: The best performance of a structure is obtained by finding the optimal material and shape connectivity for a structure within a specified region. For continuum structures this could mean to optimally place a given amount of material in a given domain to form a structure with outer boundaries and inner openings. For

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truss type structures it could mean adding/removing members between joints. Topology optimization is particularly useful in the conceptual design phase (Figure 2a–b).

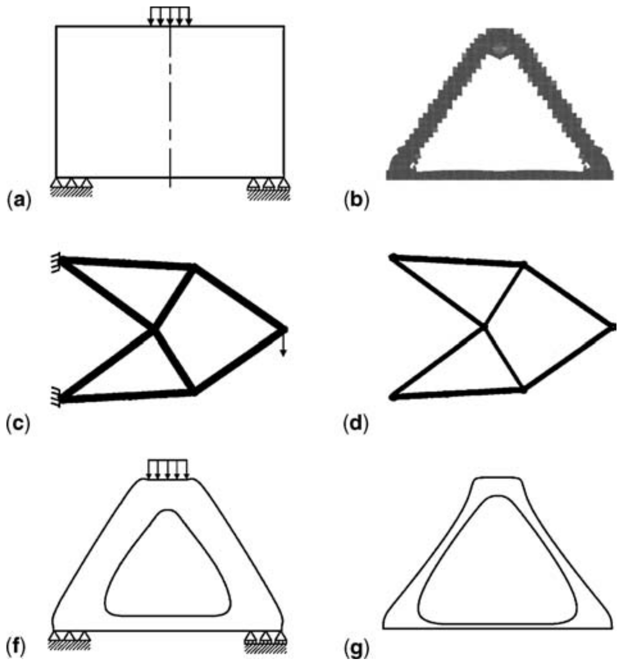

Figure 2. Structural optimization. Topology optimization for a continuum structure: (a) Original layout; (b) optimized topology. Size optimization for a discrete structure; (c) original layout; (d) optimized structure. Shape optimization for a continuum structure: (f) original layout, (g) optimized shape.

- **Size optimization**: The best performance of a parameterized structure is obtained by updating geometrical variables, such as the thickness of a continuum structure, or the diameter of the members of a discrete structure. Size optimization is generally used to find the optimal cross-sectional area of beams and stringers, or the thickness of plates and shells (see Composite Laminate Optimization with Discrete Variables) (Figure 2c–d).
- **Shape optimization**: The best performance of a structure is obtained by looking for the best shape of the domain by simply modifying the existing boundaries of the initial domain (Allaire, 2007). Shape optimization is, for instance, used to optimize the shape of a given (or suggested) rib structure that would give the minimal mass under some specified loading conditions and constraints (Figure 2f–g).

Aircraft components are structurally often designed based on stability requirements. In this case, the use of the FEM for the optimization of such components usually follows a two-phase design process. First, an initial design is obtained by performing topology optimization. This is then followed by detailed sizing/shape optimization techniques, where stability, stress and manufacture constraints can easily be included (see Section 5.5).

2.1 Topology optimization

For structures, topology optimization involves searching for the optimal connectivity within a given design space. For continuum structures this typically means adding/removing/redistributing material within a specified region generating layouts with outer boundaries and inner openings, whereas for truss structures the problem consists in finding the optimum configuration and spatial sequence of members and joints.

This section exemplifies the method for continuum structures, with focus on the material distribution method. In this method we are interested in the optimal distribution of a given material in space by determining which points of space should be material points and which points should be voids.

Mathematically, the problem is formulated as follows. Given a design domain $\Omega \subseteq \mathbb{R}^d$ $(d = 2, 3)$, a classical problem of topology optimization consists of finding the subdomain $\Omega_{\mathrm{mat}} \subseteq \Omega$ that yields the minimum compliance of the structure, that is, the minimum value of the external work, assumed as a measure of the global rigidity of the structure. Other objective functions can also be used in engineering applications, such as the weight, the cost, or the maximum stress in a point. The design domain Ω can then be described in terms of the characteristic function $\chi = \chi(\boldsymbol{x})$ that is equal to one at the points of the reference space that should be material points, and is equal to zero at those points that should be voids (no material). Denoting by $\boldsymbol{t}$ the traction force applied on a fixed part Γ_{N} of the boundary of the domain Ω and by $\boldsymbol{g}$ the specified displacement along the boundary Γ_{D}, the classical topology optimization problem can then be formulated as

Find $\chi(\boldsymbol{x})$ for $\boldsymbol{x} \in \Omega$ such that minimize

$$\int_{\Gamma_{\mathrm{N}}} \boldsymbol{t} \cdot \boldsymbol{u} \, \mathrm{d}s \quad \text{(Compliance)}$$

subject to the constraint on the volume

$$\int_{\Omega} \chi(\boldsymbol{x}) \mathrm{d}\boldsymbol{x} \leq \varphi |\Omega| \tag{1}$$

with $\varphi = |\Omega_{\mathrm{mat}}|/|\Omega| \in [0, 1]$ the assigned volume fraction, $|\Omega|$ the volume of the domain Ω and $\boldsymbol{u}$ solution of the

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equilibrium equation

$$\int_{\Omega} \boldsymbol{\sigma}(\boldsymbol{u}) : \boldsymbol{\varepsilon}(\tilde{\boldsymbol{u}})\mathrm{d}\boldsymbol{x} = \int_{\Gamma_N} \boldsymbol{t}\cdot\tilde{\boldsymbol{u}}\,\mathrm{d}s \quad \text{for any } \tilde{\boldsymbol{u}} = 0 \text{ on } \Gamma_N$$

$$\boldsymbol{u} = \boldsymbol{g} \text{ on } \Gamma_D$$

$$\boldsymbol{\sigma}(\boldsymbol{u}) = \chi \boldsymbol{D} : \boldsymbol{\varepsilon}(\boldsymbol{u})$$

where $\boldsymbol{D}$ is the material stiffness and $\boldsymbol{\varepsilon}(\boldsymbol{u})$ is the strain.

Equation (1) is an integer optimization problem that can be transformed into a continuous optimization problem (Allaire, 2007) using either the homogenization method (Bendsøe and Kikuchi, 1988; Hassani and Hinton, 1999) or the penalization method (Bendsøe and Sigmund, 2003; Zhou and Rozvany, 1991) or the relaxation method (Strang and Kohn, 1986) or some heuristic methods, such as the evolutionary method (Xie and Steven, 1997; Hinton and Sienz, 1995). Regardless the optimization method used to find the optimum layout of the structure, the result is usually a distribution of material density (see Figure 3) in terms of a grey scale geometrical representation of the structure, where typically black is assigned to a finite element when $\chi = 1$, that is, it is solid, white when $\chi = 0$, that is, it is void, and the different grey intensity for $0 < \chi < 1$. The optimal topology is afterwards discerned using some kind of image processing or intuitively using engineering judgement. In the following, only the homogenization and the penalization method will be described, referring to the cited literature for other approaches, such as those mentioned above and further methods for topology optimization, for example, level set methods (Wang, Wang and Guo, 2003).

Following original development of the classical formulation as given in equation (1), it is now possible and common practice to employ more commonly used engineering design specifications, such as, weight minimization subject to stress constraints.

2.1.1 *Topology optimization by homogenization*

Materials with a spatially varying microstructure (e.g., Figure 4b) can be introduced to describe the varying material properties in space. That is, at the microscopic level, the material model is considered to have a periodic structure with unit cell Y made of voids ($\chi = 0$) and the given material with its elasticity tensor $\boldsymbol{D}$ ($\chi = 1$). At the macroscopic level, the material model is then described by a homogeneous material with

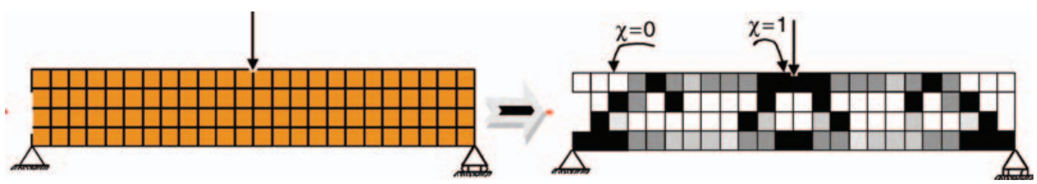

Figure 3. Topological design using the material distribution method.

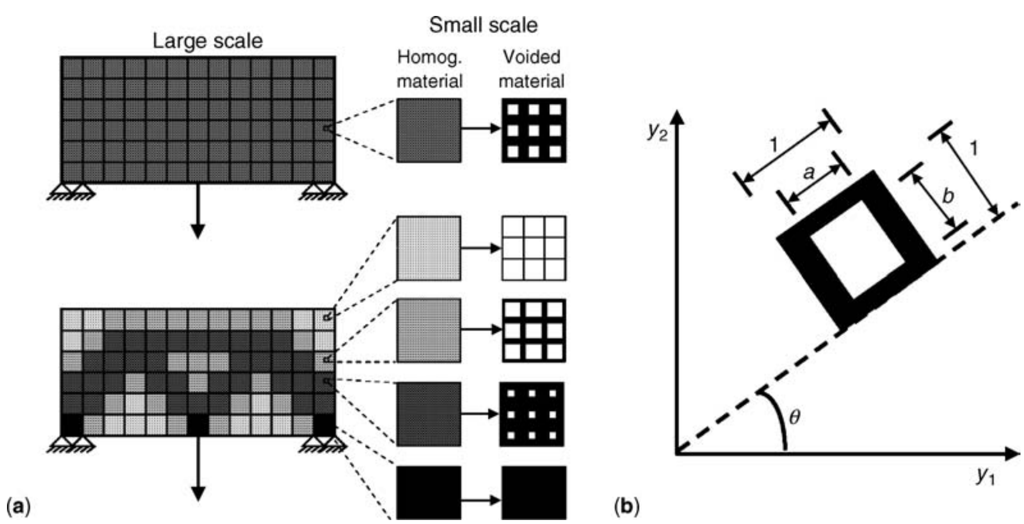

Figure 4. Topology optimization by distribution of a pre-defined type of composite material: (a) global problem; (b) unit cell Y with definition of the geometric variables (a,b,θ).

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density $0 \leq \rho \leq 1$ and homogenized elasticity tensor $\boldsymbol{D}^{\mathrm{H}}$. To exemplify the method, consider the fixed reference domain $\Omega \subseteq \mathbb{R}^2$ as shown in Figure 4a. The design variables of the optimization problem are, for the chosen *periodic* microstructure depicted in Figure 4b, the geometric variables a, b, and θ. The (pseudo-) density of the unit cell will then be given by $\rho = 1 - ab$ with $\rho \in [0, 1]$ where $\rho = 0$ and $\rho = 1$ represent the state of void and solid, respectively.

By using the FEM for the solution of the equilibrium equation (1), the type of interpolation functions for the design variable fields $(a(\boldsymbol{x}), b(\boldsymbol{x}), \theta(\boldsymbol{x}))$ and for the homogenized displacement field $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x})$ needs to be selected. In one of the early implementations of this concept, realized in Bendsøe and Kikuchi (1988), it is assumed that each finite element has a specific cellular type of microstructure. This means that in this case the field of the design variables is approximated by piecewise constant functions $\boldsymbol{\Lambda} = (\boldsymbol{\Lambda}_1, \ldots, \boldsymbol{\Lambda}_{\mathrm{N}})$ with $\boldsymbol{\Lambda}_{\mathrm{i}} = (a_i, b_i, \theta_i)$ and N the number of finite elements. The finite element based topology optimization in this case then can be formulated as

Find $\boldsymbol{\Lambda} = (\boldsymbol{\Lambda}_1, \ldots, \boldsymbol{\Lambda}_{\mathrm{N}})$ such that minimize

$$f(\boldsymbol{\Lambda}) = \boldsymbol{F}^{\mathrm{T}}\boldsymbol{U}_0 \quad \text{(Compliance)} \tag{2}$$

subject to the constraints

$$o \leq a \leq 1 \quad o \leq b \leq 1$$
$$\sum_{i=1}^{N} P_i V_i \leq \varphi V \quad \text{with} \quad P_i = (1 - a_i b_i)$$

and $\boldsymbol{U}$ solution of the discrete homogenized equilibrium equation

$$\boldsymbol{P}(\boldsymbol{\sigma}) - \boldsymbol{F} = 0$$
$$\boldsymbol{\sigma} = \boldsymbol{D}^{\mathrm{H}} : \boldsymbol{\varepsilon}(\boldsymbol{U})$$
$$\boldsymbol{U} = \boldsymbol{g} \text{ on } \Gamma_{\mathrm{D}}$$

with $\boldsymbol{D}^{\mathrm{H}}$ evaluated at the Gauss point of the finite element.

A numerical algorithm based on the optimality criteria of equation (2) has been proposed by Hassani and Hinton (1999). Inherent in the homogenization method, the structural analysis requires a two-step analysis: the solution of the cell problem in order to evaluate the effective elasticity tensor $\boldsymbol{D}^{\mathrm{H}}$ for each FE, and the solution of the homogenized problem. An interactive on-line MATLAB implementation of an FEM solution based topology optimization formulation for the case of compliance minimization of statically loaded structures can be found in Sigmund (2001).

2.1.2 *Topology optimization by penalization*

The penalization approach introduces a penalty in order to replace the integer variables of equation (2) with continuous values. This is for instance, realized by the so-called solid isotropic material with penalization model (SIMP) (Bendsøe and Sigmund, 2003) that introduces a continuous variable $\rho \in [0, 1]$ that resembles the density of a fictitious material of stiffness tensor $\boldsymbol{D}$ defined as follows

$$\boldsymbol{D}(\rho) = \rho^p \boldsymbol{D}^o \tag{3}$$

with $\boldsymbol{D}^0$ the stiffness tensor of the solid material and p the penalization factor (see Figure 5 for a graphical representation of this function). The shape of this function helps to achieve the goal of finding a design consisting mainly of void or solid areas by penalizing intermediate solutions. The optimal topology problem is therefore converted into an optimization problem on a fixed domain Ω as follows

Find $\rho(\boldsymbol{x})$ for $\boldsymbol{x} \in \Omega$ such that minimize

$$\int_{\Gamma_{\mathrm{N}}} \boldsymbol{t} \cdot \boldsymbol{u} \,\mathrm{d}s \quad \text{(Compliance)} \tag{4}$$

subject to the constraints

$$\int_{\Omega} \rho(\boldsymbol{x}) \mathrm{d}\boldsymbol{x} \leq \varphi \boldsymbol{V} \qquad 0 \leq \rho \leq 1$$

and $\boldsymbol{u}$ solution of the equilibrium equation

$$\int_{\Omega} \boldsymbol{\sigma}(\boldsymbol{u}); \boldsymbol{\varepsilon}(\tilde{\boldsymbol{u}}) \mathrm{d}\boldsymbol{x} = \int_{\Gamma_{\mathrm{N}}} t \cdot \tilde{\boldsymbol{u}} \mathrm{d}s \text{ for any } \tilde{\boldsymbol{u}} = 0 \text{ on } \Gamma_{\mathrm{N}}$$
$$\boldsymbol{u} = \boldsymbol{g} \text{ on } \Gamma_{\mathrm{D}}$$
$$\boldsymbol{\sigma} = \rho^p \boldsymbol{D}^o : \boldsymbol{\varepsilon}(\boldsymbol{u})$$

The density bound, $\rho_{\min}$, is introduced to avoid a singularity in the FE equations whereas a sufficiently big value of p, that is, $p \geq 3$, makes the stiffness tensor less than proportional to ρ. Considering an FE discretization of the reference domain with N FEs, the density ρ is approximated as element-wise constant and represented by the vector: $\boldsymbol{\rho} = \rho_1, \ldots, \rho_{\mathrm{N}}$. The discretized topology optimization problem becomes

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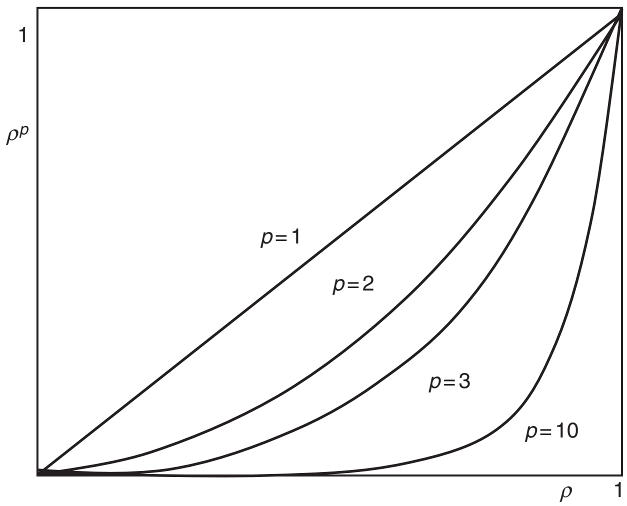

Figure 5. Graphical representation of the function ρ^p for different values of the penalization factor p.

Find $\boldsymbol{\rho} = (\rho_1, \ldots, \rho_N)$ such that minimize

$$f(\boldsymbol{\rho}) = \boldsymbol{U}^{\mathrm{T}}\boldsymbol{F} = \text{Compliance} \tag{5}$$

subject to the constraints

$$\sum_{i=1}^{N} V_i \rho_i \leq \varphi V \quad 0 \leq \rho_{\min} \leq \rho_i \leq 1$$

and $\boldsymbol{U}$ solution of the equilibrium equation

$$\left(\sum_{i=1}^{N} \rho_i^p \boldsymbol{K}_i\right)\boldsymbol{U} = \boldsymbol{F}$$

where $\boldsymbol{K}_i$ is the (global level) stiffness matrix of the element i. For other forms of penalization, see Allaire and Kohn (1993). The heuristic iterative procedure, proposed in Bendsøe and Sigmund (2003), to solve such a problem transforms the first order optimality conditions of equation (5) into a fixed-point equation of the form

$$\rho_i = \rho_i (B_i)^\eta \quad \text{for} \quad i = 1, \ldots N \tag{6}$$

with $\boldsymbol{B}_i = -(\partial f / \partial \rho_i)/\alpha V_i$, α the Lagrange multiplier associated with the volume constraint and, $\frac{\partial f}{\partial \rho_i} = -p\rho_i^{p-1}\boldsymbol{U}_i^{\mathrm{T}}\boldsymbol{K}_i\boldsymbol{U}_i$ the sensitivity of the objective function. Note that an optimum is reached when $B_i = 1$ and $\rho_{\min} < \rho_i < 1$. The numerical procedure is described in equation (7), and Figure 6 shows a flow chart of the structural topology optimization procedure.

(a) $k = 0$ $\rho_{\mathrm{e}}^k = 0.5$ for $e = 1, \ldots N$
$j = 0$ $\alpha^j = 1 = 10\,000$ m "move limit" with $m = 0.2$ as suggested value

(b) For $e = 1, \ldots N$, evaluate $\rho_{\mathrm{e}}^{k+1} = \rho_{\mathrm{e}}^k (B_{\mathrm{e}}^k)^\eta$ with $B_{\mathrm{e}}^k = \frac{-p(\rho_{\mathrm{e}}^k)^{p-1}\boldsymbol{U}_{\mathrm{e}}^T\boldsymbol{K}_{\mathrm{e}}\boldsymbol{U}_{\mathrm{e}}}{\alpha^j V_{\mathrm{e}}}$

$$\rho_{\mathrm{e}}^{k+1} = \begin{cases} \max(\rho_{\min}, \rho_{\mathrm{e}} - m) & \text{if } \rho_{\mathrm{e}}^{k+1} \leq \max(\rho_{\min}, \rho_{\mathrm{e}}^k - m) \\ \rho_{\mathrm{e}}^{k+1} & \text{if } \max(\rho_{\min}, \rho_{\mathrm{e}}^k - m) \\ & < \rho_{\mathrm{e}}^{k+1} < \min(1, \rho_{\mathrm{e}}^k + m) \\ \min(1, \rho_{\mathrm{e}}^k + m) & \text{if } \min(1, \rho_{\mathrm{e}}^k + m) \leq \rho_{\mathrm{e}}^{k+1} \end{cases} \tag{7}$$

(c) If $\sum_{\varepsilon=1}^{N} \rho_{\mathrm{e}}^{k+1} V_{\mathrm{e}} > \varphi V$
$\alpha^j = \alpha^j/2$, $j = j + 1$, GO TO (b)

(d) Compare ρ_{e}^{k+1} with ρ_{e}^k TO STOP
Otherwise SET $k = k + 1$ and GO TO (b)

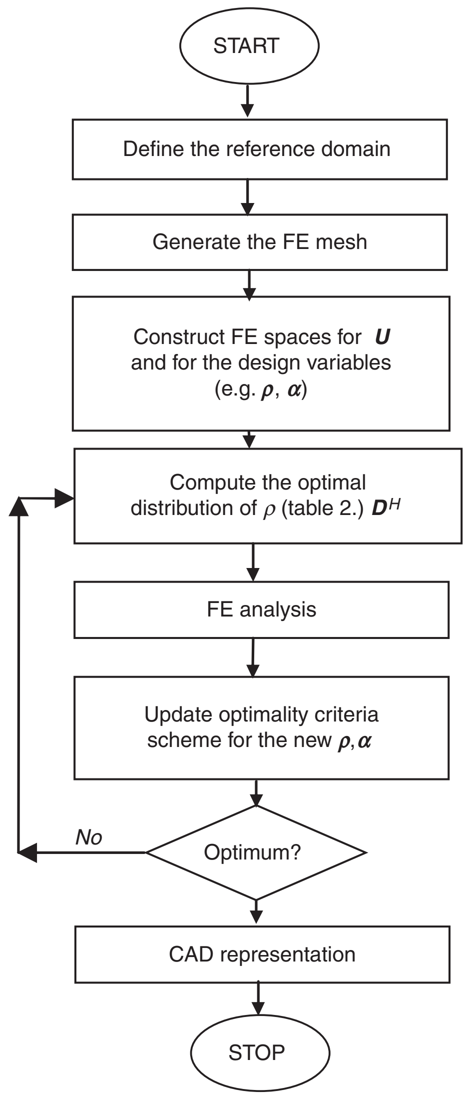

Figure 6. Flow charts of the numerical algorithms for topology optimization problems.

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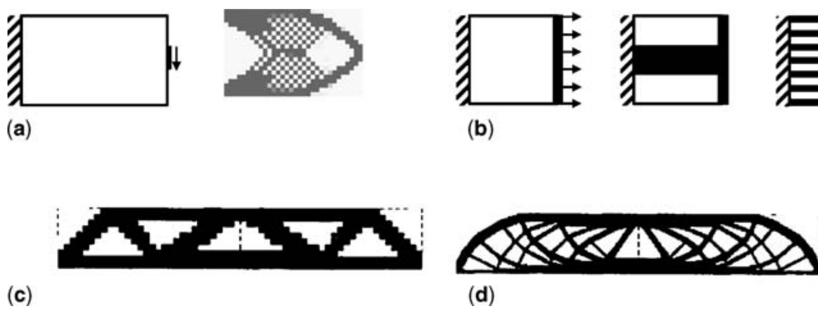

Figure 7. Typical numerical instabilities in finite dimensional topology optimization: (a) Checkerboard. Reproduced from Hassani and Hinton (1999) © Springer-Verlag; (b) local minima Reproduced from Sigmund and Petersson (1998) © Springer-Verlag; (c) Reproduced from Sigmund and Petersson (1998) © Springer-Verlag and (d) mesh dependence for 600 and 5400 elements, respectively. Reproduced from Sigmund and Petersson (1998) © Springer-Verlag.

2.1.3 *Numerical instabilities in finite element based topology optimization*

The finite dimensional topology optimization problem, in general, can be solved using different methods of nonlinear programming such as: optimality criteria methods (e.g., SIMP method in Section 2.1.2), sequential linear programming methods, or other methods for constrained optimization (see Review of Optimization Techniques). However, one must be aware of the common numerical instabilities that appear in topology optimization. These can manifest as (see Figure 7):

- *checkerboard*: formation of regions of alternating solid and void elements ordered in a checkerboard fashion;
- *mesh dependence*: qualitatively different solutions for different mesh-size or discretizations;
- *local minima*: different solutions to the same discretized problem when choosing different methods.

Different methods have been proposed to cure these problems. Among others, these are: the use of higher-order FE for the displacement function to avoid the checkerboard problem (Hassani and Hinton, 1999); the perimeter control method (Haber, Jog and Bendsoe, 1996), which limits the number of holes that can appear in a domain; filtering techniques, which limit the variation of the densities that appear in the set of admissible stiffness tensors (Sigmund and Petersson, 1998), relaxation by homogenization method to avoid mesh dependency (Bendsøe and Kikuchi, 1988), and the continuation method to avoid local minima (Allaire and Kohn, 1993).

2.2 Size optimization

This type of structural engineering design optimization problem is the best exemplified by the design of the optimal thickness distribution of an elastic plate that occupies a domain Ω with boundary $\partial\Omega$ and yields the minimum weight, for such structures or the cross section optimization of trusses. Concentrating on a plate, let $h(\boldsymbol{x})$ denote the plate thickness representing the design variable, a size design optimization problem can be then formulated as follows (Allaire, 2007).

Find $h(\boldsymbol{x})$ for $\boldsymbol{x} \in \Omega$ such that minimize

$$\int_\Omega g\rho \, \mathrm{d}\boldsymbol{x} \text{ (Weight)} \tag{8}$$

subject to the constraints

$$\sigma_\nu < \sigma^*$$

$$\int_\Omega h(\boldsymbol{x})\mathrm{d}\boldsymbol{x} = h_0|\Omega| \quad h_{\min} \leq h(\boldsymbol{x}) \leq h_{\max}$$

with h_0 a given average thickness and $\boldsymbol{u}$ solution of the static equilibrium equation

$$\int_\Omega h(\boldsymbol{x})\boldsymbol{\sigma}(\boldsymbol{u}) : \boldsymbol{\varepsilon}(\tilde{\boldsymbol{u}})\mathrm{d}\boldsymbol{x} = \int_\Omega \boldsymbol{t} \cdot \tilde{\boldsymbol{u}}\mathrm{d}\boldsymbol{x} \quad \text{for any } \tilde{\boldsymbol{u}} = 0 \text{ on } \partial\Omega$$
$$\boldsymbol{\sigma} = \boldsymbol{D} : \boldsymbol{\varepsilon}(\boldsymbol{u})$$

In equation (8), ρ denotes the material density, g the gravitational acceleration, σ_ν the von Mises equivalent stress, which is a function of the current stress state, whereas σ^* is the prescribed maximum stress.

Size optimization is widely used in the aerospace industry, where $h(\boldsymbol{x})$ can represent not only the structure's thickness, but also the frame height, the stringer height, or the cross section area. For continuous composite material structures, sizing optimization is used to determine the thickness of each layer. Additionally, the ply angle should be optimized for such structures (see Composite Laminate Optimization with Discrete Variables for further details). The problem given above is generally augmented by additional constraints, which are

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related to the control of displacement, buckling, fatigue, flutter, and manufacturing constraints.

For stability requirements one can have constraints in the form of

$$\lambda_1 > 1 \tag{9}$$

where λ_1 is the smallest positive eigenvalue for compression-loaded structures, and is given by solving the following generalized eigenvalue problem:

$$(\boldsymbol{K} - \lambda \boldsymbol{K}_{\mathrm{G}})\boldsymbol{U} = 0 \tag{10}$$

where $\boldsymbol{K}$ denotes the stiffness matrix of the whole structure or of the structural component, according to whether a global or local buckling analysis is carried out, respectively, $\boldsymbol{K}_{\mathrm{G}}$ the geometric stiffness matrix, λ an eigenvalue, and $\boldsymbol{U}$ the corresponding eigenvector. The matrix $\boldsymbol{K}_{\mathrm{G}}$ is a component of the global stiffness matrix $\boldsymbol{K}$ that arises from the nonlinear form of the strain-displacement equations (Zienkiewicz and Taylor, 2005; Bazant and Cedolin, 1991). If $\boldsymbol{K}_{\mathrm{G}}$ is replaced by the mass matrix, equation (10) represents the undamped free vibration equation and equation (9) would correspond to a constraint on the frequency.

It is worth noting that when a large number of design variables and constraints are involved in the optimization process, such as for the size optimization of the skin, stringers, frames, longerons, and doors of the rear fuselage shell structure shown in Figure 8, extremely large computational resources are required (Stettner and Schuhmacher, 2004). In this case, the large optimization problem can be broken into a series of smaller problems but this involves some approximations. The decomposition process identifies groups of design variables and constraints that interact closely with each other within the same group, but interact weakly with the rest of the design variables. In this case, the original problem can be broken down into a series of problems that can be solved independently (for further details on the topic refer to Haftka and Gürdal, 1992).

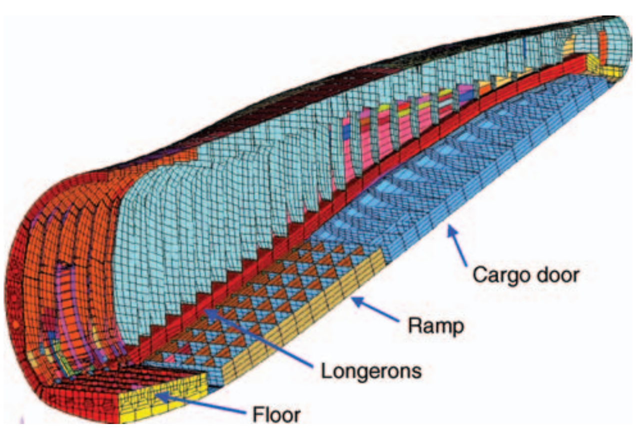

Figure 8. Finite element model of a rear fuselage shell structure. Reproduced with permission from Stettner and Schuhmacher (2004) © Altair Engineering, Ltd.

2.3 Shape optimization

Shape optimization can be defined as an optimal design problem where the design variable is the shape of the domain Ω_{mat}. To better illustrate the problem at hand, let an initial domain Ω with volume V_0 and boundary $\partial\Omega = \Gamma \cup \Gamma_{\mathrm{D}} \cup \Gamma_{\mathrm{N}}$ be given. The part of the boundary Γ is the one that can vary, whereas Γ_{D} and Γ_{N} are respectively where displacements $\boldsymbol{u}$ and traction forces $\boldsymbol{f}$ are prescribed. Using weight minimization as an example, the shape optimization problem can then be formulated as follows:

Find $\Omega_{\mathrm{mat}} \subset \Omega$ such that minimize

$$\int_{V_{\mathrm{mat}}} g\rho\, \mathrm{d}V \quad \text{(Weight)} \tag{11}$$

with $V(\Omega_{\mathrm{mat}}) = V_0$ and $\boldsymbol{u}$ the solution of the equilibrium equation

$$\int_{\Omega_{\mathrm{mat}}} \boldsymbol{\sigma}(\boldsymbol{u}) : \boldsymbol{\varepsilon}(\tilde{\boldsymbol{u}})\mathrm{d}\boldsymbol{x} = \int_{\Gamma_{\mathrm{N}}} \boldsymbol{t} \cdot \tilde{\boldsymbol{u}}\mathrm{d}s \text{ for any } \tilde{\boldsymbol{u}} = 0 \text{ on } \Gamma_{\mathrm{D}}$$
$$\boldsymbol{u} = \boldsymbol{g} \text{ on } \Gamma_{\mathrm{D}}$$
$$\boldsymbol{\sigma} = \boldsymbol{D} : \boldsymbol{\varepsilon}(\boldsymbol{u})$$

where Ω_{mat} is obtained by only "moving" the free boundary Γ. Note that one of the main difficulties here is that the domain Ω_{mat} is variable. The most common approaches proposed in the literature to solve shape optimization problems are (see Figure 9): the boundary parameterization method (BPM) and the Hadamard boundary variation method (Sokolowski and

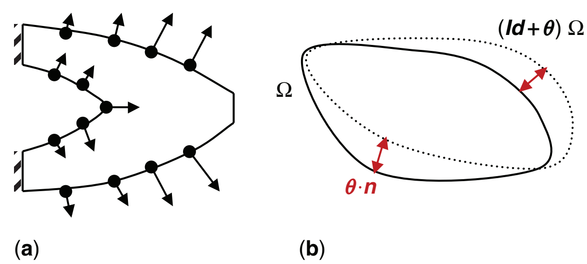

Figure 9. Shape optimization approaches: (a) Boundary parameterization method; (b) Hadamard boundary variation method.

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Zolesio, 1992) or the so-called continuous based differentiation approach.

2.3.1 *Boundary parameterization method (BPM)*

In the BPM the first step is to discretize the domain Ω into a finite number of elements. The boundary is therefore characterized by the mesh nodes, which are called control nodes and will be moved in the optimization process. Common practice is not to use all the boundary nodes in order to describe or modify the shape of a given structure, but an interpolation of the control nodes using splines, polynomial functions, and so on, (for further details, see Airfoil/Wing Optimization). Since a set of real parameters $\boldsymbol{z}$ are needed to describe such functions, for example, the position of the control nodes of splines, such parameters are used as the design variables of the shape optimization problem and equation (11) can be written as follows:

Find $\boldsymbol{z} \in \mathbb{R}^d$ such that minimize

$$f(z) = \sum_{i=1}^{N} gV_i\rho_i = \text{Weight} \tag{12}$$

with: $V(z, \boldsymbol{X}(z)) = V_0$ and $\boldsymbol{U}(z)$ the FE solution of the discrete state equations over $\Omega_{\text{mat}}(\boldsymbol{z})$

$$\boldsymbol{K}(\boldsymbol{X})\boldsymbol{U} = \boldsymbol{F}$$

where, $\boldsymbol{X}(z)$ gives the position of the FE nodes related to the position of the boundary control nodes.

A design sensitivity analysis (see Sensitivity Analysis) is then performed by differentiating the discrete form of the structural governing equations (12).

2.3.2 *Hadamard boundary variation method*

The Hadamard boundary variation method, on the other hand, is based on the idea that the boundary moves along its normal and the admissible domain Ω is parametrized in terms of a given class of functions. Considering an initial smooth open set $\Omega \subset \mathbb{R}^d$ for $d = 2, 3$ and a map $\boldsymbol{\theta}$: $\Omega \rightarrow \mathbb{R}^d$, the domain Ω_{mat} in equation (11) can be expressed as

$$\Omega_{\text{mat}} = (Id + \boldsymbol{\theta})\Omega \tag{13}$$

where for a small vector field $\boldsymbol{\theta}$, $(Id + \boldsymbol{\theta})$ is an admissible deformation of Ω. The domain Ω_{mat} can be now interpreted as an image of a one-to-one mapping of Ω and equation (11) can be reformulated as follows

Find $\boldsymbol{\theta}$ such that minimize

$$\int_{\mathrm{V}} g\rho \mathrm{d}V \text{ (Weight)}$$

with $V((Id + \boldsymbol{\theta})\Omega) = V_0$ and $\boldsymbol{u}$ solution of the equilibrium equation

$$\begin{gathered}
\int_{(Id+\boldsymbol{\theta})\Omega} \boldsymbol{\sigma}(\boldsymbol{u}) : \boldsymbol{\varepsilon}(\tilde{\boldsymbol{u}})\mathrm{d}\boldsymbol{x} = \int_{\Gamma_{\mathrm{N}}} \boldsymbol{t} \cdot \tilde{\boldsymbol{u}}\mathrm{d}s \quad \text{for any } \tilde{\boldsymbol{u}} = 0 \text{ on } \Gamma_{\mathrm{N}} \\
\boldsymbol{u} = \boldsymbol{g} \text{ on } \Gamma_{\mathrm{D}} \\
\boldsymbol{\sigma} = \boldsymbol{D} : \boldsymbol{\varepsilon}(\boldsymbol{u})
\end{gathered} \tag{14}$$

In view of the application of the gradient method (see Review of Optimization Techniques) to solve equation (14), the notion of shape derivative $f'(\Omega)$ at Ω is introduced as

$$f((Id + \boldsymbol{\theta})(\Omega)) = f(\Omega) + f'(\Omega)\boldsymbol{\theta} + o(\boldsymbol{\theta}) \tag{15}$$

where $\lim_{n\to 0} \frac{|o(\boldsymbol{\theta})|}{||\boldsymbol{\theta}||} = 0$ and $f'(\Omega)\boldsymbol{\theta}$ is called the directional derivative of f in direction of $\boldsymbol{\theta}$. By the Hadamard structure theorem (Sokolowski and Zolesio, 1992), $f'(\Omega)\boldsymbol{\theta}$ is a scalar quantity defined on the boundary Γ that depends only on the normal trace $\boldsymbol{\theta}{\cdot}\boldsymbol{n}$ as follows

$$f'(\Omega_0)\boldsymbol{\theta} = \int_{\Gamma_0} (\boldsymbol{t}{\cdot}\boldsymbol{u})(\boldsymbol{\theta}{\cdot}\boldsymbol{n})\mathrm{d}s \tag{16}$$

A physical interpretation of equation (16) is that the domain needs to be reduced (i.e., $\boldsymbol{\theta} \cdot \boldsymbol{n} < 0$) to minimize the compliance. For more details on the subject, the reader is referred to Allaire (2007).

The discrete equation (12) and the discrete form of equation (14) – obtained, for instance, by introducing an FE approximation of $\boldsymbol{\theta}$ - have the form of a classical finite dimensional constraint optimization problem (see Formulating Design Problems as Optimization Problems) and can therefore be solved by standard techniques of mathematical programming. Mathematical programming consists basically of the calculation of the objective function value and its gradients with respect to the design variables for a feasible solution (i.e., sensitivity analysis, see Sensitivity Analysis) and the calculation of a locally feasible change of the design variables. These two steps are repeated until a local minimum is reached. A flow chart of the shape optimization procedure with adaptivity, as proposed in Sienz and Hinton (1995, 1997), is then depicted in Figure 10.

In the course of a shape optimization process, the design may change considerably and the initial domain discretization (FE mesh) can lead to non-optimal designs. Adaptive procedures are then necessary to adapt the domain discretization to the current state in the optimization process. This

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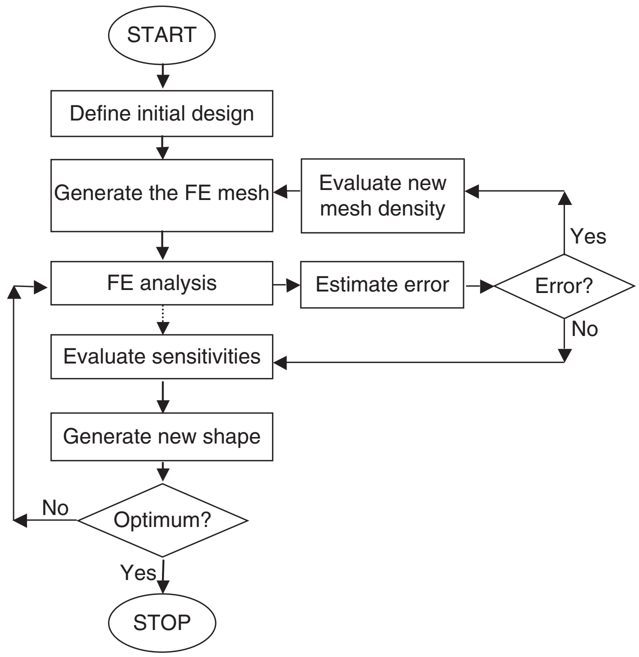

Figure 10. Flow charts of the numerical algorithms for shape optimization with adaptivity.

includes both the adaptation of the discretization of the shape to the optimum structure and the adaptation of the discretization of the state variables (such as displacements) to the structural response.

2.3.3 Algorithmic details in shape optimization

Several algorithmic difficulties can be met when running a shape optimization problem, such as:

- *oscillating boundaries*, which can be solved by regularizing the mesh, that is, smoothing the mesh at each iteration;
- *singularities on the displacements*, usually at the shape corners or changes of boundary conditions, for which the shape gradient can be set to zero near the corners;
- *volume constraint oscillations*, when the volume constraint is not exactly enforced before convergence.

3 GEOMETRY MODELING AND GRID GENERATION

Engineering design optimization develops solutions that are at the limits as defined by the constraints. It is therefore essential to ensure that the underlying analysis is accurate and it correctly represents the behavior of the optimized structure – error estimates together with adaptive mesh generation can be used to achieve this.

By applying the FEM, the continuous constrained optimization problem is transformed into a finite dimensional one by introducing an FE approximation of the continuous variables (see Formulating Design Problems as Optimization Problems). The construction of such approximations requires a partition of the continuous physical domain $\Omega \subset \mathbb{R}^d$ for $d = 2$, 3 into simpler geometric elements, which is called mesh, whereas the process of constructing the mesh is called mesh generation.

The main objective of a mesh generation procedure is to obtain a good quality mesh, that is, the mesh conforms to the geometry of the physical problem one wishes to model, and it also delivers the best possible numerical accuracy with the least number of elements. In general, this is obtained with grids composed of elements of appropriate sizes, possibly varying throughout the domain, and being of good quality shape as given by a quantitative definition of the quality of a mesh (Loehner, 2008; Frey and George, 2000).

The mesh quality measures account indirectly for the parameters that influence the accuracy of the discrete solution whereas a-posteriori error estimates provide an estimate of the error of the discrete solution in terms of only known quantities, that is, mesh element size and shape, problem data and the computed discrete solution. If one denotes by $\boldsymbol{T_h}$ a mesh of size h on Ω, $\boldsymbol{u}$ the exact solution, $\boldsymbol{u_h}$ the discrete solution, and $||\boldsymbol{u} - \boldsymbol{u_h}||$ the distance between the two functions, that is, the exact error, one says that $\boldsymbol{e}(\boldsymbol{T_h}, \text{problem data}, \boldsymbol{u_h})$ is an a-posteriori estimate of the error if a bound of the following type, called reliability estimate, holds

$$||\boldsymbol{u} - \boldsymbol{u_h}|| \le \boldsymbol{e}(\boldsymbol{T_h}, \textbf{\textit{problem data}}, \boldsymbol{u_h}) \qquad (17)$$

with $\boldsymbol{e}(\boldsymbol{T_h}, \textbf{\textit{problem data}}, \boldsymbol{u_h})$ required, for instance, to approach zero for $h \to 0$ and $\boldsymbol{u_h} \to \boldsymbol{u}$. By iterating such process, one can then envisage a mesh adaptive algorithm, which consists of successive loops (Morin, Nochetto and Siebert, 2002; Carstensen and Orlando, 2005). Sienz *et al*. (1999) integrated adaptive FEM into an overall shape optimization algorithm (see Figure 10) ensuring that the optimal design solution is based on accurate analyses.

For further details on error estimation and adaptive mesh generation the reader is referred to Error Estimation and Quality Control and Adaptive Mesh Generation and Visualization.

4 STRUCURAL ANALYSIS BY THE FINITE ELEMENT METHOD

The performance of the structure and the constraint functions limiting the design in an engineering design optimization loop is typically evaluated using the FEM. The discrete

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equations of the FEM are obtained by using FE interpolations of the displacement and strain in the form

$$\boldsymbol{u} = \boldsymbol{N}\boldsymbol{U} \quad \text{and} \quad \boldsymbol{\varepsilon}(\boldsymbol{u}) = \boldsymbol{B}\boldsymbol{U} \tag{18}$$

with $\boldsymbol{U}$ the time dependent nodal displacement vector, $\boldsymbol{N}$ the matrix of the element shape functions and $\boldsymbol{B}$ the one containing the derivative of the shape functions.

The discrete form of the balance of momentum over the domain Ω, is written then as follows

$$\boldsymbol{M}\ddot{\boldsymbol{U}} + \boldsymbol{P}(\boldsymbol{\sigma}) - \boldsymbol{F} = 0 \tag{19}$$

where $\boldsymbol{M} = \sum_e \boldsymbol{M}^{(e)}$, $\boldsymbol{P} = \sum_e \boldsymbol{P}^{(e)}$ and $\boldsymbol{F} = \sum_e \boldsymbol{F}^{(e)}$. The element arrays are defined by

$$\boldsymbol{M}^{(e)} = \int_{\Omega e} \boldsymbol{N}^{\mathrm{T}} \rho \boldsymbol{N}\, \mathrm{d}\Omega \; \boldsymbol{P}^{(e)} = \int_{\Omega e} \boldsymbol{B}^{\mathrm{T}} \boldsymbol{\sigma} \mathrm{d}\boldsymbol{\Omega} \quad \text{and}$$

$$\boldsymbol{F}^{(e)} = \int_{\Omega e} \boldsymbol{N}^{\mathrm{T}} \boldsymbol{b}\, \mathrm{d}\Omega + \int_{\Gamma_{Ne}} \boldsymbol{N}^{\mathrm{T}} \boldsymbol{t}\, \mathrm{d}\Gamma \tag{20}$$

with Ω_e the element domain, $\Gamma_{Ne} = \partial\Omega_e \cap \Gamma_N$ the part of the boundary where the external traction forces $\boldsymbol{t}$ are prescribed and $\boldsymbol{b}$ is the body forces vector.

In the case of linear elasticity the constitutive equations is given by $\boldsymbol{\sigma} = \boldsymbol{D}$e, with $\boldsymbol{D}$ the material modulus tensor, obtaining the elemental stress force

$$\boldsymbol{p}^{(e)} = \left(\int_{\Omega e} \boldsymbol{B}^{\mathrm{T}} \boldsymbol{D} \boldsymbol{B}\, \mathrm{d}\Omega \right) \boldsymbol{U} = \boldsymbol{K}^{(e)} \boldsymbol{U} \tag{21}$$

with $\boldsymbol{K}^{(e)}$ is the global element level stiffness matrix. For more details on this topic the reader is referred to Fundamentals of Discretization Methods, Finite Element Analysis of Composite Plates and Shells, Meshfree Discretization Methods for Solid Mechanics, Extended Finite Element Methods, Error Estimation and Quality Control, Adaptive Mesh Generation and Visualization, Computational Methods in Buckling and Instability, Thermal Analysis, Computational Dynamics.

5 NUMERICAL EXAMPLES

5.1 Influence of optimization problem definition on outcome

The optimal design of a given square plate is here analyzed by slightly modifying the original definition of the problem. The initial design is shown in Figure 11 where an orthogonal tensile load with a ratio of 2:1 on opposite sides is applied to the plate. Note that only a symmetric quarter of the plate is modelled. For the base line optimal design, there are five design variables, which can move along radial lines to modify the internal boundary. The objective is to minimize the volume of the structure subject to a constraint on the equivalent stress. Figure 12 contains the base line optimal design and also the different solutions that can be obtained by modifying one parameter of the base line optimization problem.

The second problem looks at the volume minimization of a connecting rod subject to a limit on the equivalent stress as given in Figure 13. The aim is to demonstrate the influence of the analysis parameters and design parameters on the optimal design. There is a tensile load modelled as a linearly varying line load in the bold hole. The design model makes use of quarter symmetry. Figure 14 shows the best solutions obtained employing different FE models, while Figure 15 shows the best solution obtained for this problem. The importance of employing an accurate underlying FEM for engineering design optimization is highlighted when comparing Figure 14a with Figure 14b. Although the final volume is lower for the former and the stress constraint seems to be satisfied, the former solution is infeasible: a subsequent adaptive

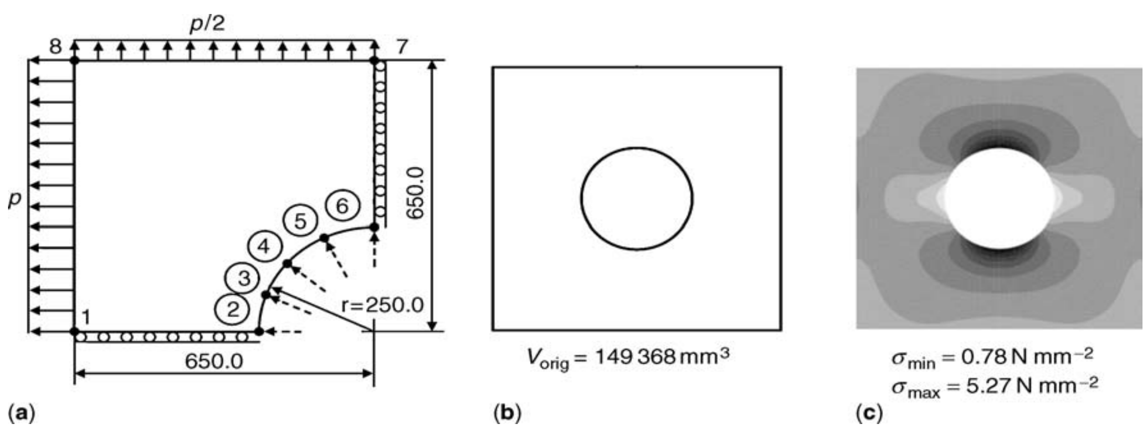

Figure 11. (a) Shape optimization problem definition of square plate with five design variables; (b) original square plate geometry; and (c) equivalent stress distribution.

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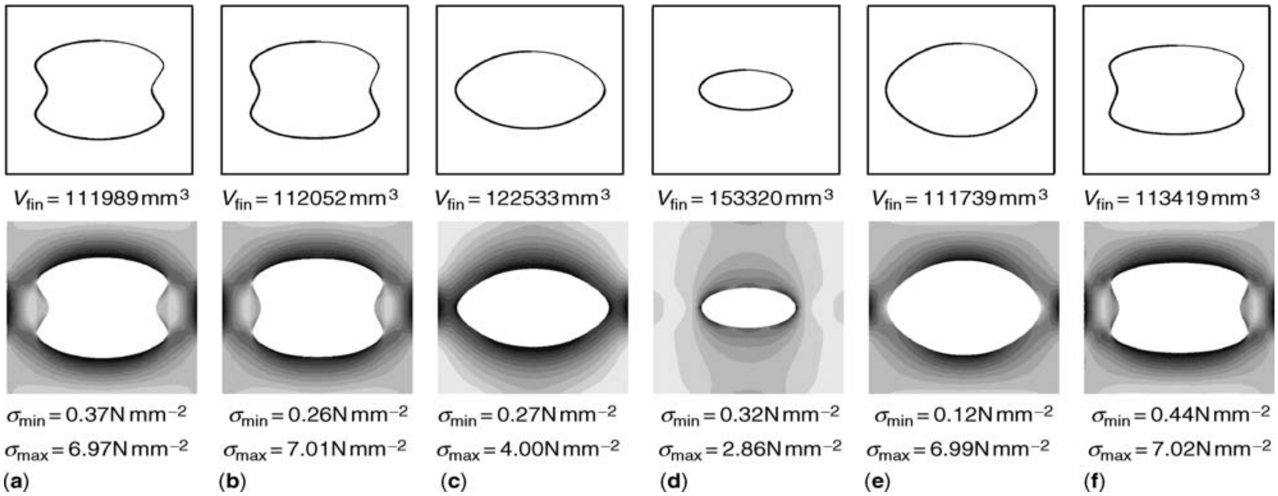

Figure 12. Various solutions to the square plate problem obtained by modifying one parameter of the baseline optimization problem. (a) Base line optimal design; (b) starting geometry modified to a central square cut-out, rotated by 45°; (c) reduced stress constraint; (d) stress levelling on the surface of the circular cut-out as objective function; (e) 3 radial design variables; (f) loading modified to 3:1 ratio.

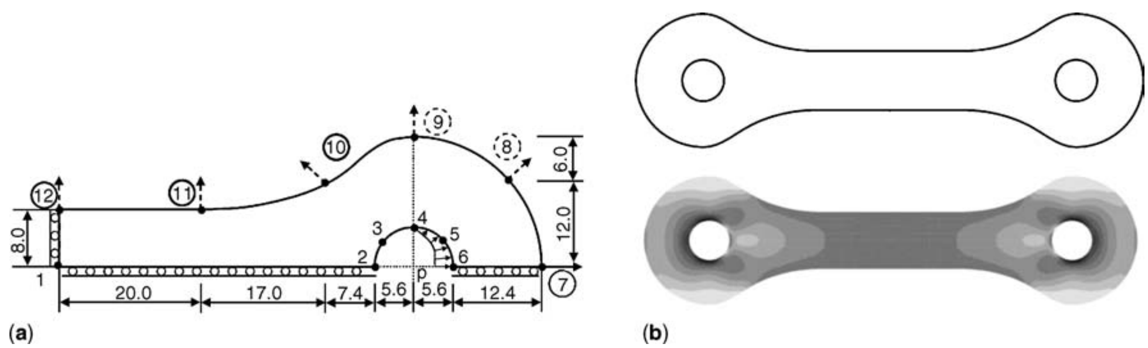

Figure 13. (a) Shape optimization problem definition of a connecting rod with five design variables, (b) original square plate geometry and equivalent stress distribution.

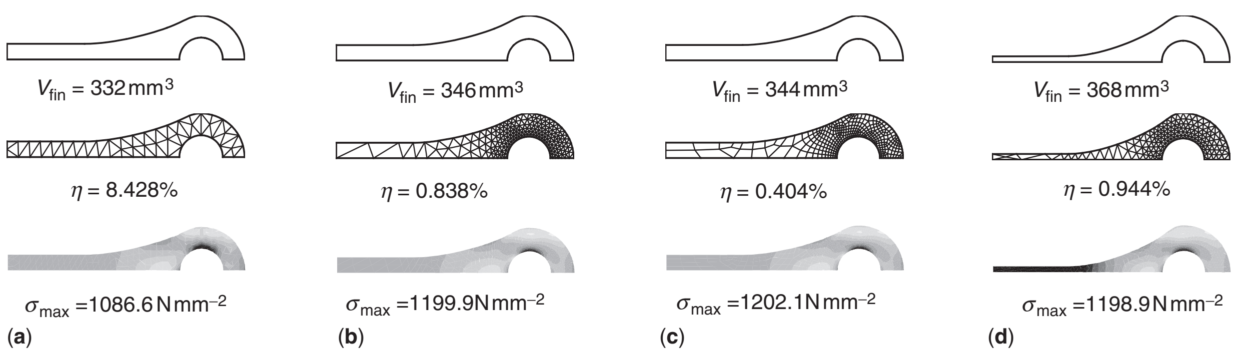

Figure 14. Various solutions to the connecting rod problem obtained by adjusting analysis model parameters or a design constraint. (a) Base line optimal design; (b) with accurate, adaptive FEM; (c) with accurate, adaptive FEM and quadrilateral elements; (d) with accurate, adaptive FEM and relaxed side constraint.

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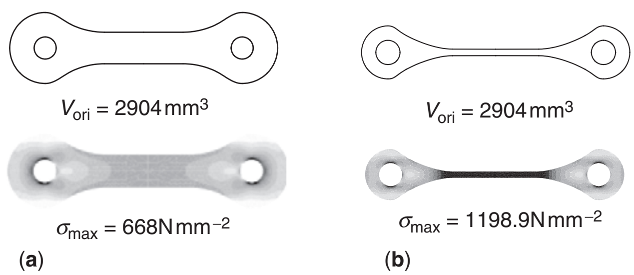

Figure 15. (a) Initial design of the connecting rod; and (b) best optimal solution.

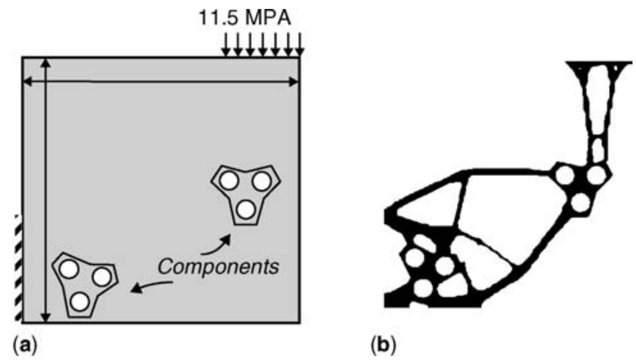

Figure 16. Topology optimization of an aircraft structure with fixed components. (a) Initial design. (b) Optimum design (example produced with Hyperworks v9.1).

FEM of the final shape revealed a 7% violation of the stress constraint.

5.2 Topology optimization

In this example topology optimization by the penalization method is employed to find the optimum supporting structure of an aircraft, which interconnects a pair of fixed, solid components for the design domain shown in Figure 16a. The optimization problem is formulated as a volume minimization problem with a constraint on the compliance value. Once the design with a minimum volume is obtained and due to the presence of grey zones, a final "cleaning up" of the optimized model is performed by choosing the elements with a material density higher than 40%. The optimum design is finally depicted in Figure 16*b*. Large scale problems currently solved in the aerospace industry have two million FEs and include up to 40 load cases while minimizing the structural mass subject to stress and buckling constraints.

5.3 Size optimization

A preliminary design of a composite hat-stiffened skin panel is considered here, it is an upper cover of a typical passenger bay in a BWB transport airplane. This panel is made of laminated composite material with identical stiffeners running in the x direction, as shown in Figure 17a and as inspired by the work done by Vitali *et al*. (2002). The loading conditions assumed for the panel are: an internal pressure and a compression along the x direction. A linear elastic orthotropic material is assumed for the panel. Considering the thicknesses t_s and t_f, t_w, t_c as design variables (see Figure 17b), a size optimization problem can then be formulated using the panel mass as objective function, together with stress and buckling constraints.

Buckling modes for the initial and optimum designs are shown in Figure 18, depicting in detail the location and the deformation of the structure for the lowest buckling mode. The buckling occurs only in the web of the profile on the symmetry plane. Localization of the instabilities can be observed in both designs. Nevertheless, the optimum design shows a qualitative change of the buckling mode with respect to the initial design shown in Figure 18a.

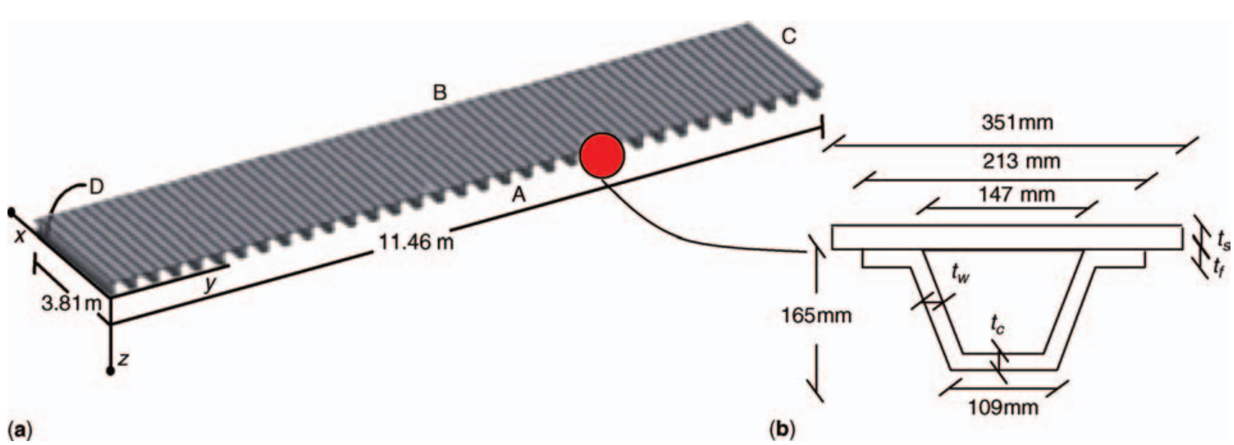

Figure 17. (a) Hat-stiffened skin panel; and (b) single module design variables.

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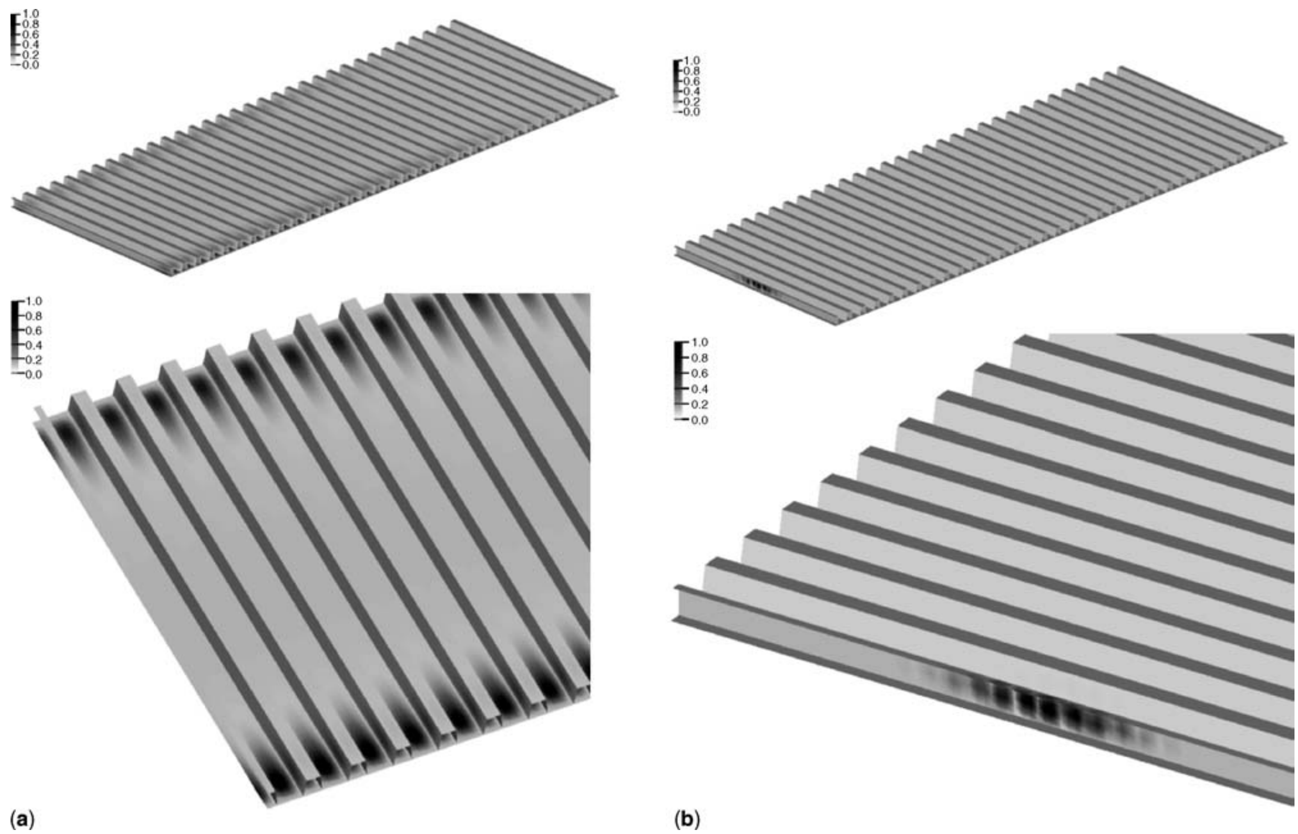

Figure 18. (a) Buckling modes contours for the initial; and (b) optimized panel.

5.4 Shape versus topology optimization

The classical, so-called "MBB beam" problem is considered here in order to compare the BPM shape optimization method and topology optimization by the penalization method. The optimization problem is first formulated as the minimization of the structure's volume with a maximum vertical displacement and von Mises stress as constraints. Due to symmetry, computations are performed in only one half of the domain.

The initial domain and the boundary conditions used for the topology optimization are shown in Figure 19a. An optimum layout of the structure with intermediate dense elements is obtained, which is subsequently enhanced by discarding the elements with a density lower than a certain value. This leads to the optimum design shown in Figure 19c.

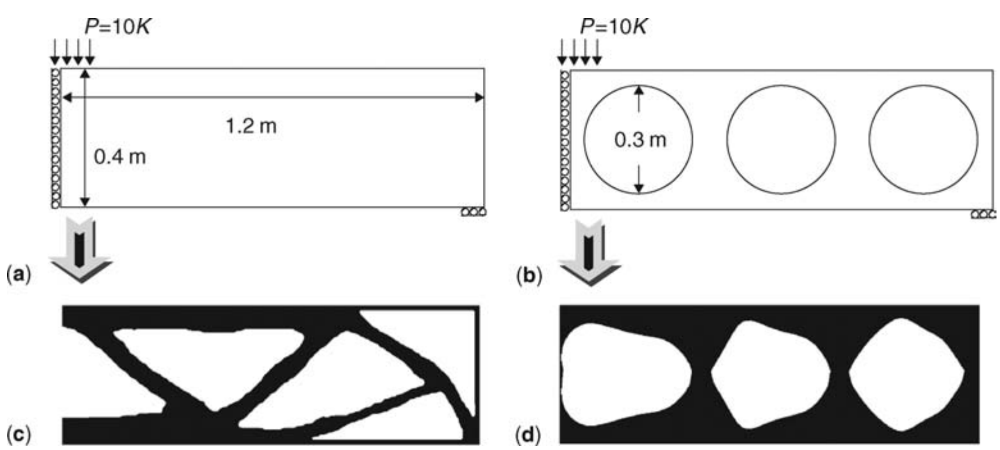

Figure 19. Initial and final design by means of topology (a,c) and shape (b,d) optimization (example produced with Hyperworks v9.1).

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Applying the BPM shape optimization method to the initial design shown in Figure 19b, with the control points of B-splines governing the shape of the holes chosen as design variables, the optimal design shown in Figure 19d is obtained.

Both designs meet the same displacement and stress constraints. However, it can be observed that, the volume of the structure obtained with the topology optimization method is considerably less than the one obtained with the shape optimization method. This is due to the ability of the former methodology to find the optimum load path of a given structure, but with a non-smooth definition of the boundary curves as drawback. Since the shape optimization method allows finer details of the boundary to be controlled, it is thus common practice to integrate both methods. This is shown in the next two examples.

5.5 Fully integrated engineering design optimization

5.5.1 Rib design using topology, shape, and size optimization techniques

The design of the leading edge droop nose rib shown in Figure 20a is a complex process as a large number of design variables are involved. Upscaling previous design solutions proved to be unsatisfactory. Therefore, the topology optimization method was selected as the first step in the design process to give a rational basis for a designer to select the initial domain for subsequent size and shape optimization. In this first step, the objective function is chosen as the compliance. The optimal load paths of Figure 20b are then obtained with considerable mass reductions. Since the image obtained from the topology optimization design is not very distinct, image extraction techniques are used and the geometric model of Figure 20c is obtained. For the size/shape optimization step, the design variables are chosen as the height/thickness of the vertical stiffeners, and the thickness of the horizontal segments. The rib's mass is taken as an objective function, with both stability and stress constraints, together with a reduction factor for fatigue applied to the von Mises allowable stress. The results are depicted in Figure 20d with a further reduction in mass.

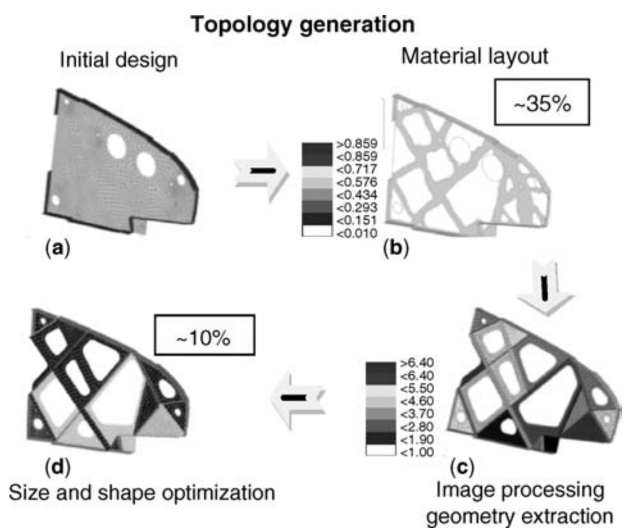

Figure 20. Fully integrated design optimization of an aircraft component. Reproduced with permission from Krog *et al.* (2004) © Altair Engineering, Ltd.

5.5.2 Fly-wheel design using topology optimization and shape optimization with adaptivity

This section describes the fully integrated design optimization process of a fly-wheel structure, which includes mesh adaptivity based on a-posteriori error estimation of the FE solution. The fly-wheel is subjected to a centrifugal load, as volume force, resulting from its rotation, and a Neumann boundary condition over the external boundary. The optimization process starts with the initial design shown in Figure 21a. The optimum layout of the fly-wheel is obtained by applying topology optimization. Considering a set volume fraction, the fly-wheel's normalized compliance is considerably reduced and the structure's layout, shown in Figure 21b, is found. Next, the set of key points depicted in Figure 21c are identified and used for the boundary representation of a new design with a cubic B-spline interpolation. Shape optimization with adaptive FE is then performed starting from the initial mesh depicted in Figure 21d. Adopting the total mass of the fly-wheel as objective function, together with the maximum principal stress value and a maximum error of the FE solution as constraints, the final design shown in Figure 21e is obtained.

6 CONCLUDING REMARKS

This chapter has introduced continuous optimization built around the three-column concept. Depending on the selection of the design variables, the mathematical formulation of the problem, the parameterization and the available optimization algorithms, different types of optimization problems have been illustrated. Finally, a set of examples highlighting achievements and also difficulties have been carefully developed with the aim of illustrating the practical application of such techniques.

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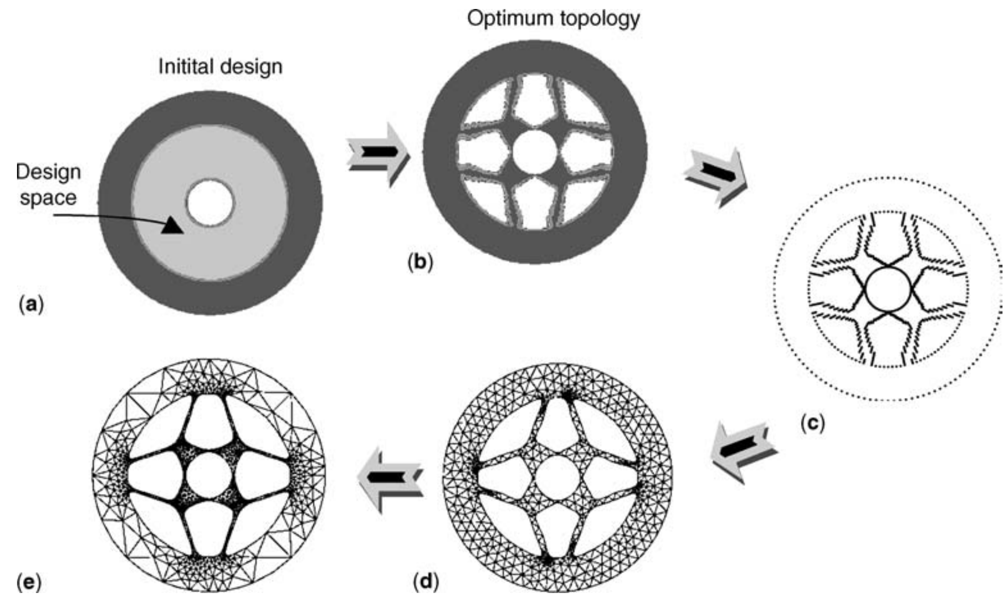

Figure 21. Fully integrated design optimization of a fly-wheel. (a) Initial design; (b) optimum topology; (c) boundary detection; (d) Initial mesh for shape optimization; (e) Optimum shape w/mesh adaptivity. Reproduced with permission from Sienz *et al.* (1999) © Saxe-Coburg Publications.

The topology optimization method gives a rational basis for a designer to select the initial domain. This is due to its ability to find the layout for a structure, but with a non-smooth definition of the boundary curves as drawback. Size and shape optimization techniques instead allow already existing boundaries to be controlled in more detail. Fully integrated methods, where topology, size, and shape optimization techniques are included, are common practice in the aerospace industry. Accuracy of the discrete solution of the underlying analysis is essential; error estimates together with adaptive mesh generation can be used to achieve this.

Most of the concepts described above can be found in commercially available computer-aided engineering (CAE) tools commonly used in the industry, such as ALTAIR HYPERWORKS, ANSYS, and NASTRAN. Typically, they have a wide range of tools within one software framework or workbench to solve large scale optimization problems by combining performance data management, process automation, good data exchange facilities with robust, reliable meshing tools and general purpose, accurate FE solvers for structures, fluids, thermal, acoustic, electromagnetic, and/or multiphysics problems. Various optimization technologies for automated, optimal engineering design complement the CAE tools, using topology, size or shape methods, or a combination of them, to simultaneously satisfy objectives and meet constraint targets for stiffness, strength, durability, crashworthiness, noise and vibration, mass, cost, manufacturability, and reliability. These powerful tools for solving engineering design problems with increasing complexity are very helpful for lesser-experienced design engineers, but exploiting their full potential needs extensive experience so that the results are always optimal and feasible solutions.

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RELATED CHAPTERS

Fundamentals of Discretization Methods
Finite Element Analysis of Composite Plates and Shells
Meshfree Discretization Methods for Solid Mechanics
Extended Finite Element Methods
Error Estimation and Quality Control
Adaptive Mesh Generation and Visualization
Computational Methods in Buckling and Instability
Thermal Analysis
Computational Dynamics
Computational Optimization
Formulating Design Problems as Optimization Problems
Review of Optimization Techniques
Composite Laminate Optimization with Discrete Variables
Airfoil/Wing Optimization
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