TEMA 1 - A

TRANSFORMACIÓN DE LAPLACE

Definición

$$\mathcal{L}[f(t)] = F(s) = \int_0^\infty e^{-st} f(t)\,dt$$

Propiedades

1. Linealidad

$$\mathcal{L}[a\,f(t) + b\,g(t)] = a\,\mathcal{L}[f(t)] + b\,\mathcal{L}[g(t)]$$

2. Tranformación de funciones trasladadas

$$\mathcal{L}[f(t-L)] = e^{-Ls}\,\mathcal{L}[f(t)]$$

3. Tranformación de la diferenciación real

$$\mathcal{L}\left[\frac{df(t)}{dt}\right] = s\,F(s) - f(0)$$

$$\mathcal{L}\left[\frac{d^2 f(t)}{dt^2}\right] = s^2\,F(s) - s\,f(0) - \left.\frac{df}{dt}\right|_{t=0}$$

4. Transformación de la integral temporal

$$\mathcal{L}\left[\int_0^t f(\tau)\,d\tau\right] = \frac{F(s)}{s}$$

5. Teorema del valor inicial

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty}[sF(s)]$$

6. Teorema del valor final

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0}[sF(s)]$$

f(t)	**F(s)**
$\delta(t)$ (Impulso unitario)	1
1 (escalón unitario)	$\frac{1}{s}$
t (rampa unitaria)	$\frac{1}{s^2}$
t^{n-1}	$\frac{(n-1)!}{s^n}$
e^{-at}	$\frac{1}{s+a}$
$\frac{1}{t} e^{-t/t}$	$\frac{1}{t s + 1}$
$\frac{1}{t^n (n-1)!} t^{n-1} e^{-t/t}$	$\frac{1}{(t s + 1)^n}$
$\frac{1}{\tau_1 - \tau_2} (e^{-t/\tau_1} - e^{-t/\tau_2})$	$\frac{1}{(t_1 s+1)(t_2 s+1)}$
$1 - \frac{\tau_1 - \tau_3}{\tau_1 - \tau_2} e^{-t/\tau_1} + \frac{\tau_2 - \tau_3}{\tau_1 - \tau_2} e^{-t/\tau_2}$	$\frac{t_3 s+1}{s(t_1 s+1)(t_2 s+1)}$
$1 - e^{-t/t}$	$\frac{1}{s(t s + 1)}$
$1 + \frac{1}{\tau_1 - \tau_2} (\tau_2 e^{-t/\tau_2} - \tau_1 e^{-t/\tau_1})$	$\frac{1}{s(t_1 s + 1)(t_2 s + 1)}$
$1 - \frac{(\tau + t)}{\tau} e^{-t/\tau}$	$\frac{1}{s(\tau s + 1)^2}$
$\text{sen}(\omega t)$	$\frac{w}{s^2 + w^2}$
$\cos(\omega t)$	$\frac{s}{s^2 + \omega^2}$
$e^{-at}\ \text{sen}(\omega t)$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$e^{-at} \cos(\omega t)$	$\frac{s+a}{(s+a)^2 + \omega^2}$
$\frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\ \text{sen}(\omega_n \sqrt{1-\xi^2}\, t)$	$\frac{1}{\frac{1}{\omega_n^2} s^2 + \frac{2\xi}{\omega_n} s + 1}$
$1 + \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\ \text{sen}(\omega_n \sqrt{1-\xi^2}\, t + \phi)$ $\text{tg}\,\phi = \frac{\sqrt{1-\xi^2}}{-\xi}$ $\quad 0 \le \xi < 1$	$\frac{1}{s(\frac{1}{\omega_n^2} s^2 + \frac{2\xi}{\omega_n} s + 1)}$
$t\, (e^{-t/t} + \frac{t}{t} - 1)$	$\frac{1}{s^2 (t s + 1)}$
$\frac{\omega\tau}{\tau^2\omega^2 + 1} e^{-t/\tau} + \frac{1}{\sqrt{\tau^2\omega^2 + 1}} \text{sen}(\omega t + \phi)$ $\text{tg}\phi = -\tau\omega$	$\frac{\omega}{(\tau s + 1)(s^2 + \omega^2)}$